Rubi 4.16.0.4 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions"

Test results for the 1917 problems in "1.1.1.2 (a+b x)^m (c+d x)^n.m"

Problem 369: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\frac{b\,x^m}{2\,(a+b\,x)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x}}\right) dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x}}$$

Result (type 5, 92 leaves, 5 steps):

$$\frac{x^m\left(-\frac{b\,x}{a}\right)^{-m}\text{Hypergeometric2F1}\left[-\frac{1}{2},\,-m,\,\frac{1}{2},\,1+\frac{b\,x}{a}\right]}{\sqrt{a+b\,x}}-\frac{2\,m\,x^m\left(-\frac{b\,x}{a}\right)^{-m}\sqrt{a+b\,x}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\,1-m,\,\frac{3}{2},\,1+\frac{b\,x}{a}\right]}{a}$$

Test results for the 3201 problems in "1.1.1.3 (a+b x)^m (c+d x)^n (e+f x)^p.m"

Problem 957: Result valid but suboptimal antiderivative.

$$\int (e\,x)^m\,(a-b\,x)^{2+n}\,(a+b\,x)^n\,dx$$

Optimal (type 5, 211 leaves, ? steps):

$$-\frac{(e x)^{1+m} (a-b x)^{1+n} (a+b x)^{1+n}}{e (3+m+2 n)}+\frac{2 a^2 (2+m+n) (e x)^{1+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{1+m}{2},-n,\frac{3+m}{2},\frac{b^2 x^2}{a^2}\right]}{e (1+m) (3+m+2 n)}-\frac{2 a b (e x)^{2+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{2+m}{2},-n,\frac{4+m}{2},\frac{b^2 x^2}{a^2}\right]}{e^2 (2+m)}$$

Result (type 5, 238 leaves, 11 steps) :

$$\frac{a^2 (e x)^{1+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{1+m}{2},-n,\frac{3+m}{2},\frac{b^2 x^2}{a^2}\right]}{e (1+m)}-\frac{2 a b (e x)^{2+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{2+m}{2},-n,\frac{4+m}{2},\frac{b^2 x^2}{a^2}\right]}{e^2 (2+m)}+\frac{b^2 (e x)^{3+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{3+m}{2},-n,\frac{5+m}{2},\frac{b^2 x^2}{a^2}\right]}{e^3 (3+m)}$$

Problem 1001: Result unnecessarily involves higher level functions.

$$\int \frac{(1-a x)^{1-n} (1+a x)^{1+n}}{x^2} \, dx$$

Optimal (type 5, 106 leaves, 3 steps) :

$$-\frac{2 a (1-a x)^{1-n} (1+a x)^{-1+n} \text{Hypergeometric2F1}\left[2,1-n,2-n,\frac{1-a x}{1+a x}\right]}{1-n}+\frac{2^n a (1-a x)^{1-n} \text{Hypergeometric2F1}\left[1-n,-n,2-n,\frac{1}{2} (1-a x)\right]}{1-n}$$

Result (type 6, 48 leaves, 1 step) :

$$\frac{2^{1-n} a (1+a x)^{2+n} \text{AppellF1}\left[2+n,-1+n,2,3+n,\frac{1}{2} (1+a x),1+a x\right]}{2+n}$$

Problem 1006: Result valid but suboptimal antiderivative.

$$\int \frac{(a-b x)^{-n} (a+b x)^{1+n}}{x} \, dx$$

Optimal (type 5, 142 leaves, 6 steps) :

$$\frac{(a-b x)^{1-n} (a+b x)^n}{2 n}-\frac{a (a-b x)^{-n} (a+b x)^n \text{Hypergeometric2F1}\left[1, n, 1+n, \frac{a+b x}{a-b x}\right]}{n}+\frac{2^{-1-n} (1+2 n) (a-b x)^{-n} \left(\frac{a-b x}{a}\right)^n (a+b x)^{1+n} \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{a+b x}{2 a}\right]}{n (1+n)}$$

Result (type 5, 173 leaves, 7 steps):

$$\frac{a (a-b x)^{-n} (a+b x)^n \text{Hypergeometric2F1}\left[1, -n, 1-n, \frac{a-b x}{a+b x}\right]}{n}-\frac{2^n a (a-b x)^{-n} (a+b x)^n \left(\frac{a+b x}{a}\right)^{-n} \text{Hypergeometric2F1}\left[-n, -n, 1-n, \frac{a-b x}{2 a}\right]}{n}+\frac{2^{-n} (a-b x)^{-n} \left(\frac{a-b x}{a}\right)^n (a+b x)^{1+n} \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{a+b x}{2 a}\right]}{1+n}$$

Problem 1007: Result unnecessarily involves higher level functions.

$$\int \frac{(a-b x)^{-n} (a+b x)^{1+n}}{x^2} \, dx$$

Optimal (type 5, 140 leaves, 5 steps):

$$-\frac{(a-b x)^{-n} (a+b x)^{1+n}}{x}+\frac{b (1+2 n) (a-b x)^{-n} (a+b x)^n \text{Hypergeometric2F1}\left[1, -n, 1-n, \frac{a-b x}{a+b x}\right]}{n}-\frac{2^n b (a-b x)^{-n} (a+b x)^n \left(\frac{a+b x}{a}\right)^{-n} \text{Hypergeometric2F1}\left[-n, -n, 1-n, \frac{a-b x}{2 a}\right]}{n}$$

Result (type 6, 76 leaves, 2 steps):

$$\frac{2^{-n} b (a-b x)^{-n} \left(\frac{a-b x}{a}\right)^n (a+b x)^{2+n} \text{AppellF1}\left[2+n, n, 2, 3+n, \frac{a+b x}{2 a}, \frac{a+b x}{a}\right]}{a^2 (2+n)}$$

Problem 2121: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(1-2 x)^{3/2} (3+5 x)^2} \, dx$$

Optimal (type 3, 63 leaves, 4 steps):

$$\frac{6}{121 \sqrt{1-2 x}}-\frac{1}{11 \sqrt{1-2 x} (3+5 x)}-\frac{6}{121} \sqrt{\frac{5}{11}} \text{ArcTanh}\left[\sqrt{\frac{5}{11}} \sqrt{1-2 x}\right]$$

Result (type 3, 70 leaves, 4 steps):

$$\frac{2}{11\sqrt{1-2x}\,(3+5x)}-\frac{15\sqrt{1-2x}}{121\,(3+5x)}-\frac{6}{121}\sqrt{\frac{5}{11}}\,\text{ArcTanh}\left[\sqrt{\frac{5}{11}}\,\sqrt{1-2x}\right]$$

Problem 2144: Result valid but suboptimal antiderivative.

$$\int\frac{3+5x}{(1-2x)^{5/2}\,(2+3x)^4}\,dx$$

Optimal (type 3, 116 leaves, 7 steps):

$$\frac{160}{3087\,(1-2x)^{3/2}}+\frac{160}{2401\sqrt{1-2x}}+\frac{1}{63\,(1-2x)^{3/2}\,(2+3x)^3}-\frac{16}{147\,(1-2x)^{3/2}\,(2+3x)^2}-\frac{16}{147\,(1-2x)^{3/2}\,(2+3x)}-\frac{160\sqrt{\frac{3}{7}}\,\text{ArcTanh}\left[\sqrt{\frac{3}{7}}\,\sqrt{1-2x}\right]}{2401}$$

Result (type 3, 130 leaves, 7 steps):

$$\frac{1}{63\,(1-2x)^{3/2}\,(2+3x)^3}+\frac{64}{441\,(1-2x)^{3/2}\,(2+3x)^2}+\frac{64}{147\sqrt{1-2x}\,(2+3x)^2}-\frac{80\sqrt{1-2x}}{343\,(2+3x)^2}-\frac{240\sqrt{1-2x}}{2401\,(2+3x)}-\frac{160\sqrt{\frac{3}{7}}\,\text{ArcTanh}\left[\sqrt{\frac{3}{7}}\,\sqrt{1-2x}\right]}{2401}$$

Problem 2145: Result valid but suboptimal antiderivative.

$$\int\frac{3+5x}{(1-2x)^{5/2}\,(2+3x)^5}\,dx$$

Optimal (type 3, 136 leaves, 8 steps):

$$\frac{215}{9604\,(1-2x)^{3/2}}+\frac{1935}{67\,228\sqrt{1-2x}}+\frac{1}{84\,(1-2x)^{3/2}\,(2+3x)^4}-\frac{43}{588\,(1-2x)^{3/2}\,(2+3x)^3}-\frac{129}{2744\,(1-2x)^{3/2}\,(2+3x)^2}-\frac{129}{2744\,(1-2x)^{3/2}\,(2+3x)}-\frac{1935\sqrt{\frac{3}{7}}\,\text{ArcTanh}\left[\sqrt{\frac{3}{7}}\,\sqrt{1-2x}\right]}{67\,228}$$

Result (type 3, 150 leaves, 8 steps):

$$\frac{1}{84\,(1-2x)^{3/2}\,(2+3x)^4}+\frac{43}{294\,(1-2x)^{3/2}\,(2+3x)^3}+\frac{387}{686\,\sqrt{1-2x}\,(2+3x)^3}-$$

$$\frac{387\,\sqrt{1-2x}}{1372\,(2+3x)^3}-\frac{1935\,\sqrt{1-2x}}{19\,208\,(2+3x)^2}-\frac{5805\,\sqrt{1-2x}}{134\,456\,(2+3x)}-\frac{1935\,\sqrt{\frac{3}{7}}\,\text{ArcTanh}\left[\sqrt{\frac{3}{7}}\,\sqrt{1-2x}\right]}{67\,228}$$

Problem 2196: Result valid but suboptimal antiderivative.

$$\int\frac{1}{(1-2x)^{5/2}\,(3+5x)^3}\,dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\frac{35}{3993\,(1-2x)^{3/2}}+\frac{175}{14\,641\,\sqrt{1-2x}}-\frac{1}{22\,(1-2x)^{3/2}\,(3+5x)^2}-\frac{7}{242\,(1-2x)^{3/2}\,(3+5x)}-\frac{175\,\sqrt{\frac{5}{11}}\,\text{ArcTanh}\left[\sqrt{\frac{5}{11}}\,\sqrt{1-2x}\right]}{14\,641}$$

Result (type 3, 110 leaves, 6 steps):

$$\frac{2}{33\,(1-2x)^{3/2}\,(3+5x)^2}+\frac{70}{363\,\sqrt{1-2x}\,(3+5x)^2}-\frac{875\,\sqrt{1-2x}}{7986\,(3+5x)^2}-\frac{875\,\sqrt{1-2x}}{29\,282\,(3+5x)}-\frac{175\,\sqrt{\frac{5}{11}}\,\text{ArcTanh}\left[\sqrt{\frac{5}{11}}\,\sqrt{1-2x}\right]}{14\,641}$$

Problem 3075: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b\,x)^m\,(c+d\,x)^{-1-m}}{e+f\,x}\,dx$$

Optimal (type 5, 72 leaves, 1 step):

$$-\frac{(a+b\,x)^m\,(c+d\,x)^{-m}\,\text{Hypergeometric2F1}\left[1,\,-m,\,1-m,\,\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]}{(d\,e-c\,f)\,m}$$

Result (type 5, 75 leaves, 1 step):

$$\frac{(a+b\,x)^{1+m}\,(c+d\,x)^{-1-m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{(b\,e-a\,f)\,(1+m)}$$

Problem 3077: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-1-m}}{(e+f x)^3} \, dx$$

Optimal (type 5, 283 leaves, 4 steps):

$$-\frac{f (a+b x)^{1+m} (c+d x)^{-m}}{2 (b e-a f) (d e-c f) (e+f x)^2}-\frac{f (b (3 d e-c f (1-m))-a d f (2+m)) (a+b x)^{1+m} (c+d x)^{-m}}{2 (b e-a f)^2 (d e-c f)^2 (e+f x)}+$$
$$\left(\left(2 a b d f (1+m) (2 d e+c f m)-b^2 \left(2 d^2 e^2+4 c d e f m-c^2 f^2 (1-m) m\right)-a^2 d^2 f^2 \left(2+3 m+m^2\right)\right)\right.$$
$$\left.(a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]\right) \Big/ \left(2 (b e-a f)^2 (d e-c f)^3 m\right)$$

Result (type 5, 300 leaves, 4 steps):

$$-\frac{f (a d f (2+m)-b (2 d e+c f m)) (a+b x)^{1+m} (c+d x)^{1-m}}{2 (b c-a d) (b e-a f) (d e-c f)^2 m (e+f x)^2}+\frac{d (a+b x)^{1+m} (c+d x)^{-m}}{(b c-a d) (d e-c f) m (e+f x)^2}+$$
$$\left(\left(2 a b d f (1+m) (2 d e+c f m)-b^2 \left(2 d^2 e^2+4 c d e f m-c^2 f^2 (1-m) m\right)-a^2 d^2 f^2 \left(2+3 m+m^2\right)\right) (a+b x)^{1+m}\right.$$
$$\left.(c+d x)^{-1-m} \text{Hypergeometric2F1}\left[2,1+m,2+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]\right) \Big/ \left(2 (b e-a f)^3 (d e-c f)^2 m (1+m)\right)$$

Problem 3078: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-1-m}}{(e+f x)^4} \, dx$$

Optimal (type 5, 498 leaves, 5 steps):

$$-\frac{f(a+bx)^{1+m}(c+dx)^{-m}}{3(be-af)(de-cf)(e+fx)^3}-\frac{f(b(5de-cf(2-m))-adf(3+m))(a+bx)^{1+m}(c+dx)^{-m}}{6(be-af)^2(de-cf)^2(e+fx)^2}-$$
$$\left(f\left(a^2d^2f^2\left(6+5m+m^2\right)-abdf\left(de(15+8m)-cf\left(3-2m-2m^2\right)\right)+b^2\left(11d^2e^2-cdef(7-8m)+c^2f^2\left(2-3m+m^2\right)\right)\right)(a+bx)^{1+m}(c+dx)^{-m}\right)\Big/\left(6(be-af)^3(de-cf)^3(e+fx)\right)+$$
$$\Big(\Big(3ab^2df(1+m)\left(6d^2e^2+6cdefm-c^2f^2(1-m)m\right)-3a^2bd^2f^2(3de+cfm)\left(2+3m+m^2\right)+a^3d^3f^3\left(6+11m+6m^2+m^3\right)-$$
$$b^3\left(6d^3e^3+18cd^2e^2fm-9c^2def^2(1-m)m+c^3f^3m\left(2-3m+m^2\right)\right)\Big)(a+bx)^m$$
$$(c+dx)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]\Big)\Big/\left(6(be-af)^3(de-cf)^4m\right)$$

Result (type 5, 520 leaves, 5 steps):

$$-\frac{f(adf(3+m)-b(3de+cfm))(a+bx)^{1+m}(c+dx)^{1-m}}{3(bc-ad)(be-af)(de-cf)^2m(e+fx)^3}+\frac{d(a+bx)^{1+m}(c+dx)^{-m}}{(bc-ad)(de-cf)m(e+fx)^3}+$$
$$\left(f\left(b^2\left(6d^2e^2+7cdefm-c^2f^2(2-m)m\right)+a^2d^2f^2\left(6+5m+m^2\right)-abdf(cfm(3+2m)+de(12+7m))\right)(a+bx)^{1+m}(c+dx)^{1-m}\right)\Big/\left(6(bc-ad)(be-af)^2(de-cf)^3m(e+fx)^2\right)+$$
$$\Big(\Big(3ab^2df(1+m)\left(6d^2e^2+6cdefm-c^2f^2(1-m)m\right)-3a^2bd^2f^2(3de+cfm)\left(2+3m+m^2\right)+a^3d^3f^3\left(6+11m+6m^2+m^3\right)-$$
$$b^3\left(6d^3e^3+18cd^2e^2fm-9c^2def^2(1-m)m+c^3f^3m\left(2-3m+m^2\right)\right)\Big)(a+bx)^{1+m}$$
$$(c+dx)^{-1-m}\,\text{Hypergeometric2F1}\left[2,1+m,2+m,\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]\Big)\Big/\left(6(be-af)^4(de-cf)^3m(1+m)\right)$$

Problem 3084: Result valid but suboptimal antiderivative.

$$\int\frac{(a+bx)^m(c+dx)^{-2-m}}{e+fx}\,dx$$

Optimal (type 5, 120 leaves, 2 steps):

$$\frac{d(a+bx)^{1+m}(c+dx)^{-1-m}}{(bc-ad)(de-cf)(1+m)}+\frac{f(a+bx)^m(c+dx)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]}{(de-cf)^2m}$$

Result (type 5, 135 leaves, 2 steps):

$$\frac{d(a+bx)^{1+m}(c+dx)^{-1-m}}{(bc-ad)(de-cf)(1+m)}-\frac{f(a+bx)^{1+m}(c+dx)^{-1-m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af)(de-cf)(1+m)}$$

Problem 3085: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-2-m}}{(e+f x)^2} \, dx$$

Optimal (type 5, 233 leaves, 4 steps):

$$-\frac{d (a d f (2+m)-b (d e+c f (1+m))) (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c-a d) (b e-a f) (d e-c f)^2 (1+m)}-\frac{f (a+b x)^{1+m} (c+d x)^{-1-m}}{(b e-a f) (d e-c f) (e+f x)}-\frac{f (a d f (2+m)-b (2 d e+c f m)) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]}{(b e-a f) (d e-c f)^3 m}$$

Result (type 5, 243 leaves, 4 steps):

$$\frac{d (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c-a d) (d e-c f) (1+m) (e+f x)}+\frac{f (b d e+b c f (1+m)-a d f (2+m)) (a+b x)^{1+m} (c+d x)^{-m}}{(b c-a d) (b e-a f) (d e-c f)^2 (1+m) (e+f x)}+\left(f (a d f (2+m)-b (2 d e+c f m)) (a+b x)^{1+m} (c+d x)^{-1-m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]\right)\Big/\left((b e-a f)^2 (d e-c f)^2 (1+m)\right)$$

Problem 3086: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-2-m}}{(e+f x)^3} \, dx$$

Optimal (type 5, 432 leaves, 5 steps):

$$\left(d \left(a^2 d^2 f^2 \left(6+5 m+m^2\right)+b^2 \left(2 d^2 e^2+5 c d e f (1+m)-c^2 f^2 \left(1-m^2\right)\right)-a b d f \left(d e (9+5 m)+c f \left(3+5 m+2 m^2\right)\right)\right) (a+b x)^{1+m} (c+d x)^{-1-m}\right)\Big/\left(2 (b c-a d) (b e-a f)^2 (d e-c f)^3 (1+m)\right)-\frac{f (a+b x)^{1+m} (c+d x)^{-1-m}}{2 (b e-a f) (d e-c f) (e+f x)^2}-\frac{f (b (4 d e-c f (1-m))-a d f (3+m)) (a+b x)^{1+m} (c+d x)^{-1-m}}{2 (b e-a f)^2 (d e-c f)^2 (e+f x)}-\left(f \left(2 a b d f (2+m) (3 d e+c f m)-b^2 \left(6 d^2 e^2+6 c d e f m-c^2 f^2 (1-m) m\right)-a^2 d^2 f^2 \left(6+5 m+m^2\right)\right) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]\right)\Big/\left(2 (b e-a f)^2 (d e-c f)^4 m\right)$$

Result (type 5, 452 leaves, 5 steps):

$$\frac{d\,(a+b x)^{1+m}\,(c+d x)^{-1-m}}{(b c-a d)\,(d e-c f)\,(1+m)\,(e+f x)^2}+\frac{f\,(2 b d e+b c f\,(1+m)-a d f\,(3+m))\,(a+b x)^{1+m}\,(c+d x)^{-m}}{2\,(b c-a d)\,(b e-a f)\,(d e-c f)^2\,(1+m)\,(e+f x)^2}+$$
$$\left(f\,\left(a^2 d^2 f^2\,(6+5 m+m^2)+b^2\,(2 d^2 e^2+5 c d e f\,(1+m)-c^2 f^2\,(1-m^2))-a b d f\,(d e\,(9+5 m)+c f\,(3+5 m+2 m^2))\right)\,(a+b x)^{1+m}\,(c+d x)^{-m}\right)\Big/$$
$$\left(2\,(b c-a d)\,(b e-a f)^2\,(d e-c f)^3\,(1+m)\,(e+f x)\right)+$$
$$\Big(f\,\left(2 a b d f\,(2+m)\,(3 d e+c f m)-b^2\,(6 d^2 e^2+6 c d e f m-c^2 f^2\,(1-m)\,m)-a^2 d^2 f^2\,(6+5 m+m^2)\right)\,(a+b x)^{1+m}$$
$$(c+d x)^{-1-m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f)\,(a+b x)}{(b e-a f)\,(c+d x)}\right]\Big)\Big/\left(2\,(b e-a f)^3\,(d e-c f)^3\,(1+m)\right)$$

Problem 3093: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b x)^m\,(c+d x)^{-3-m}}{e+f x}\,dx$$

Optimal (type 5, 196 leaves, 4 steps):

$$\frac{d\,(a+b x)^{1+m}\,(c+d x)^{-2-m}}{(b c-a d)\,(d e-c f)\,(2+m)}+\frac{d\,(a d f\,(2+m)+b\,(d e-c f\,(3+m)))\,(a+b x)^{1+m}\,(c+d x)^{-1-m}}{(b c-a d)^2\,(d e-c f)^2\,(1+m)\,(2+m)}-$$
$$\frac{f^2\,(a+b x)^m\,(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f)\,(c+d x)}{(d e-c f)\,(a+b x)}\right]}{(d e-c f)^3\,m}$$

Result (type 5, 208 leaves, 4 steps):

$$\frac{d\,(a+b x)^{1+m}\,(c+d x)^{-2-m}}{(b c-a d)\,(d e-c f)\,(2+m)}+\frac{d\,(b d e+a d f\,(2+m)-b c f\,(3+m))\,(a+b x)^{1+m}\,(c+d x)^{-1-m}}{(b c-a d)^2\,(d e-c f)^2\,(1+m)\,(2+m)}+$$
$$\frac{f^2\,(a+b x)^{1+m}\,(c+d x)^{-1-m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f)\,(a+b x)}{(b e-a f)\,(c+d x)}\right]}{(b e-a f)\,(d e-c f)^2\,(1+m)}$$

Problem 3094: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b x)^m\,(c+d x)^{-3-m}}{(e+f x)^2}\,dx$$

Optimal (type 5, 384 leaves, 5 steps):

$$-\frac{d\,(a d f\,(3+m)-b\,(d e+c f\,(2+m)))\,(a+b x)^{1+m}\,(c+d x)^{-2-m}}{(b c-a d)\,(b e-a f)\,(d e-c f)^2\,(2+m)}-$$
$$\left(d\,\left(a^2 d^2 f^2\,(6+5 m+m^2)-b^2\,(d^2 e^2-c d e f\,(5+2 m)-c^2 f^2\,(2+3 m+m^2))-a b d f\,(d e\,(3+2 m)+c f\,(9+8 m+2 m^2))\right)\,(a+b x)^{1+m}\,(c+d x)^{-1-m}\right)\Big/$$
$$\left((b c-a d)^2\,(b e-a f)\,(d e-c f)^3\,(1+m)\,(2+m)\right)-\frac{f\,(a+b x)^{1+m}\,(c+d x)^{-2-m}}{(b e-a f)\,(d e-c f)\,(e+f x)}+$$
$$\frac{f^2\,(a d f\,(3+m)-b\,(3 d e+c f m))\,(a+b x)^m\,(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f)\,(c+d x)}{(d e-c f)\,(a+b x)}\right]}{(b e-a f)\,(d e-c f)^4\,m}$$

Result (type 5, 398 leaves, 5 steps):

$$-\left(\left(d\,\left(a^2 d^2 f^2\,(6+5 m+m^2)-b^2\,(d^2 e^2-c d e f\,(5+2 m)-c^2 f^2\,(2+3 m+m^2))-a b d f\,(d e\,(3+2 m)+c f\,(9+8 m+2 m^2))\right)\,(a+b x)^{1+m}\,(c+d x)^{-1-m}\right)\Big/\right.$$
$$\left.\left((b c-a d)^2\,(b e-a f)\,(d e-c f)^3\,(1+m)\,(2+m)\right)\right)+$$
$$\frac{d\,(a+b x)^{1+m}\,(c+d x)^{-2-m}}{(b c-a d)\,(d e-c f)\,(2+m)\,(e+f x)}+\frac{f\,(b d e+b c f\,(2+m)-a d f\,(3+m))\,(a+b x)^{1+m}\,(c+d x)^{-1-m}}{(b c-a d)\,(b e-a f)\,(d e-c f)^2\,(2+m)\,(e+f x)}-$$
$$\left(f^2\,(a d f\,(3+m)-b\,(3 d e+c f m))\,(a+b x)^{1+m}\,(c+d x)^{-1-m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f)\,(a+b x)}{(b e-a f)\,(c+d x)}\right]\right)\Big/$$
$$\left((b e-a f)^2\,(d e-c f)^3\,(1+m)\right)$$

Problem 3101: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m\,(c+d x)^{-4-m}}{e+f x}\,dx$$

Optimal (type 5, 330 leaves, 5 steps):

$$\frac{d\,(a+b x)^{1+m}\,(c+d x)^{-3-m}}{(b c-a d)\,(d e-c f)\,(3+m)}+\frac{d\,(a d f\,(3+m)+b\,(2 d e-c f\,(5+m)))\,(a+b x)^{1+m}\,(c+d x)^{-2-m}}{(b c-a d)^2\,(d e-c f)^2\,(2+m)\,(3+m)}+$$
$$\left(d\,\left(a^2 d^2 f^2\,(6+5 m+m^2)+a b d f\,(3+m)\,(d e-c f\,(5+2 m))+b^2\,(2 d^2 e^2-c d e f\,(7+m)+c^2 f^2\,(11+6 m+m^2))\right)\,(a+b x)^{1+m}\,(c+d x)^{-1-m}\right)\Big/$$
$$\left((b c-a d)^3\,(d e-c f)^3\,(1+m)\,(2+m)\,(3+m)\right)+\frac{f^3\,(a+b x)^m\,(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f)\,(c+d x)}{(d e-c f)\,(a+b x)}\right]}{(d e-c f)^4\,m}$$

Result (type 5, 344 leaves, 5 steps):

$$\frac{d\,(a+b\,x)^{1+m}\,(c+d\,x)^{-3-m}}{(b\,c-a\,d)\,(d\,e-c\,f)\,(3+m)}+\frac{d\,(2\,b\,d\,e+a\,d\,f\,(3+m)-b\,c\,f\,(5+m))\,(a+b\,x)^{1+m}\,(c+d\,x)^{-2-m}}{(b\,c-a\,d)^2\,(d\,e-c\,f)^2\,(2+m)\,(3+m)}+$$
$$\left(d\left(a^2\,d^2\,f^2\,(6+5\,m+m^2)+a\,b\,d\,f\,(3+m)\,(d\,e-c\,f\,(5+2\,m))+b^2\,(2\,d^2\,e^2-c\,d\,e\,f\,(7+m)+c^2\,f^2\,(11+6\,m+m^2))\right)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-1-m}\right)\Big/$$
$$\left((b\,c-a\,d)^3\,(d\,e-c\,f)^3\,(1+m)\,(2+m)\,(3+m)\right)-\frac{f^3\,(a+b\,x)^{1+m}\,(c+d\,x)^{-1-m}\,\text{Hypergeometric2F1}\left[1,\,1+m,\,2+m,\,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{(b\,e-a\,f)\,(d\,e-c\,f)^3\,(1+m)}$$

Problem 3102: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b\,x)^m\,(c+d\,x)^{-4-m}}{(e+f\,x)^2}\,dx$$

Optimal (type 5, 634 leaves, 6 steps):

$$-\frac{d\,(a\,d\,f\,(4+m)-b\,(d\,e+c\,f\,(3+m)))\,(a+b\,x)^{1+m}\,(c+d\,x)^{-3-m}}{(b\,c-a\,d)\,(b\,e-a\,f)\,(d\,e-c\,f)^2\,(3+m)}-$$
$$\left(d\left(a^2\,d^2\,f^2\,(12+7\,m+m^2)-b^2\,(2\,d^2\,e^2-2\,c\,d\,e\,f\,(4+m)-c^2\,f^2\,(6+5\,m+m^2))-2\,a\,b\,d\,f\,(d\,e\,(2+m)+c\,f\,(10+6\,m+m^2))\right)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-2-m}\right)\Big/$$
$$\left((b\,c-a\,d)^2\,(b\,e-a\,f)\,(d\,e-c\,f)^3\,(2+m)\,(3+m)\right)-$$
$$\Big(d\,\big(a^3\,d^3\,f^3\,(24+26\,m+9\,m^2+m^3)-a^2\,b\,d^2\,f^2\,(3+m)\,(d\,e\,(4+3\,m)+c\,f\,(20+15\,m+3\,m^2))-b^3\,(2\,d^3\,e^3-2\,c\,d^2\,e^2\,f\,(5+m)+c^2\,d\,e\,f^2\,(26+17\,m+3\,m^2)+$$
$$c^3\,f^3\,(6+11\,m+6\,m^2+m^3))-a\,b^2\,d\,f\,(2\,d^2\,e^2\,(2+m)-2\,c\,d\,e\,f\,(16+15\,m+3\,m^2)-c^2\,f^2\,(44+50\,m+21\,m^2+3\,m^3))\big)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-1-m}\Big)\Big/$$
$$\left((b\,c-a\,d)^3\,(b\,e-a\,f)\,(d\,e-c\,f)^4\,(1+m)\,(2+m)\,(3+m)\right)-\frac{f\,(a+b\,x)^{1+m}\,(c+d\,x)^{-3-m}}{(b\,e-a\,f)\,(d\,e-c\,f)\,(e+f\,x)}-$$
$$\frac{f^3\,(a\,d\,f\,(4+m)-b\,(4\,d\,e+c\,f\,m))\,(a+b\,x)^m\,(c+d\,x)^{-m}\,\text{Hypergeometric2F1}\left[1,\,-m,\,1-m,\,\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]}{(b\,e-a\,f)\,(d\,e-c\,f)^5\,m}$$

Result (type 5, 646 leaves, 6 steps):

$$-\left(\left(d\left(a^2 d^2 f^2\left(12+7 m+m^2\right)-b^2\left(2 d^2 e^2-2 c d e f(4+m)-c^2 f^2\left(6+5 m+m^2\right)\right)-2 a b d f\left(d e(2+m)+c f\left(10+6 m+m^2\right)\right)\right)(a+b x)^{1+m}(c+d x)^{-2-m}\right)\Big/\left((b c-a d)^2(b e-a f)(d e-c f)^3(2+m)(3+m)\right)\right)-$$

$$\left(d\left(a^3 d^3 f^3\left(24+26 m+9 m^2+m^3\right)-a^2 b d^2 f^2(3+m)\left(d e(4+3 m)+c f\left(20+15 m+3 m^2\right)\right)-b^3\left(2 d^3 e^3-2 c d^2 e^2 f(5+m)+c^2 d e f^2\left(26+17 m+3 m^2\right)+c^3 f^3\left(6+11 m+6 m^2+m^3\right)\right)-a b^2 d f\left(2 d^2 e^2(2+m)-2 c d e f\left(16+15 m+3 m^2\right)-c^2 f^2\left(44+50 m+21 m^2+3 m^3\right)\right)\right)(a+b x)^{1+m}(c+d x)^{-1-m}\right)\Big/$$

$$\left((b c-a d)^3(b e-a f)(d e-c f)^4(1+m)(2+m)(3+m)\right)+\frac{d(a+b x)^{1+m}(c+d x)^{-3-m}}{(b c-a d)(d e-c f)(3+m)(e+f x)}+$$

$$\frac{f(b d e+b c f(3+m)-a d f(4+m))(a+b x)^{1+m}(c+d x)^{-2-m}}{(b c-a d)(b e-a f)(d e-c f)^2(3+m)(e+f x)}+$$

$$\left(f^3(a d f(4+m)-b(4 d e+c f m))(a+b x)^{1+m}(c+d x)^{-1-m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right)\Big/$$

$$\left((b e-a f)^2(d e-c f)^4(1+m)\right)$$

Problem 3110: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b x)^m(c+d x)^{-5-m}}{e+f x}\,dx$$

Optimal (type 5, 557 leaves, 6 steps):

$$\frac{d(a+b x)^{1+m}(c+d x)^{-4-m}}{(b c-a d)(d e-c f)(4+m)}+\frac{d(a d f(4+m)+b(3 d e-c f(7+m)))(a+b x)^{1+m}(c+d x)^{-3-m}}{(b c-a d)^2(d e-c f)^2(3+m)(4+m)}+$$

$$\left(d\left(a^2 d^2 f^2\left(12+7 m+m^2\right)+2 a b d f(4+m)(d e-c f(4+m))+b^2\left(6 d^2 e^2-2 c d e f(10+m)+c^2 f^2\left(26+9 m+m^2\right)\right)\right)(a+b x)^{1+m}(c+d x)^{-2-m}\right)\Big/$$

$$\left((b c-a d)^3(d e-c f)^3(2+m)(3+m)(4+m)\right)+$$

$$\left(d\left(a^3 d^3 f^3\left(24+26 m+9 m^2+m^3\right)+a^2 b d^2 f^2\left(12+7 m+m^2\right)(d e-c f(7+3 m))+a b^2 d f(4+m)\left(2 d^2 e^2-2 c d e f(5+m)+c^2 f^2\left(26+17 m+3 m^2\right)\right)+b^3\left(6 d^3 e^3-2 c d^2 e^2 f(13+m)+c^2 d e f^2\left(46+11 m+m^2\right)-c^3 f^3\left(50+35 m+10 m^2+m^3\right)\right)\right)(a+b x)^{1+m}(c+d x)^{-1-m}\right)\Big/$$

$$\left((b c-a d)^4(d e-c f)^4(1+m)(2+m)(3+m)(4+m)\right)-\frac{f^4(a+b x)^m(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]}{(d e-c f)^5 m}$$

Result (type 5, 569 leaves, 6 steps):

$$\frac{d\,(a+b\,x)^{1+m}\,(c+d\,x)^{-4-m}}{(b\,c-a\,d)\,(d\,e-c\,f)\,(4+m)}+\frac{d\,(3\,b\,d\,e+a\,d\,f\,(4+m)-b\,c\,f\,(7+m))\,(a+b\,x)^{1+m}\,(c+d\,x)^{-3-m}}{(b\,c-a\,d)^2\,(d\,e-c\,f)^2\,(3+m)\,(4+m)}+$$
$$\left(d\,\left(a^2\,d^2\,f^2\,(12+7\,m+m^2)+2\,a\,b\,d\,f\,(4+m)\,(d\,e-c\,f\,(4+m))+b^2\,\left(6\,d^2\,e^2-2\,c\,d\,e\,f\,(10+m)+c^2\,f^2\,(26+9\,m+m^2)\right)\right)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-2-m}\right)\Big/$$
$$\left((b\,c-a\,d)^3\,(d\,e-c\,f)^3\,(2+m)\,(3+m)\,(4+m)\right)+$$
$$\left(d\,\left(a^3\,d^3\,f^3\,(24+26\,m+9\,m^2+m^3)+a^2\,b\,d^2\,f^2\,(12+7\,m+m^2)\,(d\,e-c\,f\,(7+3\,m))+a\,b^2\,d\,f\,(4+m)\,\left(2\,d^2\,e^2-2\,c\,d\,e\,f\,(5+m)+c^2\,f^2\,(26+17\,m+3\,m^2)\right)+\right.\right.$$
$$\left.\left.b^3\,\left(6\,d^3\,e^3-2\,c\,d^2\,e^2\,f\,(13+m)+c^2\,d\,e\,f^2\,(46+11\,m+m^2)-c^3\,f^3\,(50+35\,m+10\,m^2+m^3)\right)\right)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-1-m}\right)\Big/$$
$$\left((b\,c-a\,d)^4\,(d\,e-c\,f)^4\,(1+m)\,(2+m)\,(3+m)\,(4+m)\right)+\frac{f^4\,(a+b\,x)^{1+m}\,(c+d\,x)^{-1-m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{(b\,e-a\,f)\,(d\,e-c\,f)^4\,(1+m)}$$

Problem 3116: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,x)^m\,(c+d\,x)^{1-m}}{e+f\,x}\,dx$$

Optimal (type 5, 230 leaves, 6 steps):

$$-\frac{d\,(d\,e-c\,f)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}}{(b\,c-a\,d)\,f^2\,m}-\frac{(d\,e-c\,f)\,(a+b\,x)^m\,(c+d\,x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]}{f^2\,m}+\frac{1}{b\,(b\,c-a\,d)\,f^2\,m\,(1+m)}$$
$$d\,(b\,(d\,e-c\,f\,(1-m))-a\,d\,f\,m)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}\,\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\,\text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]$$

Result (type 5, 220 leaves, 7 steps):

$$\frac{(d\,e-c\,f)\,(a+b\,x)^m\,(c+d\,x)^{-m}\,\text{Hypergeometric2F1}\left[1,m,1+m,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{f^2\,m}-$$
$$\frac{(d\,e-c\,f)\,(a+b\,x)^m\,(c+d\,x)^{-m}\,\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\,\text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{f^2\,m}+$$
$$\frac{d\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}\,\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\,\text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b\,f\,(1+m)}$$

Problem 3117: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b\,x)^m\,(c+d\,x)^{1-m}}{(e+f\,x)^2}\,dx$$

Optimal (type 5, 190 leaves, 6 steps):

$$-\frac{(a+b x)^m (c+d x)^{1-m}}{f (e+f x)}+\frac{(a d f (1-m)-b (d e-c f m)) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,m,1+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]}{f^2 (b e-a f) m}+$$
$$\frac{d (a+b x)^m (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d (a+b x)}{b c-a d}\right]}{f^2 m}$$

Result (type 6, 108 leaves, 2 steps):

$$\frac{(b c-a d) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m,-1+m,2,2+m,-\frac{d (a+b x)}{b c-a d},-\frac{f (a+b x)}{b e-a f}\right]}{(b e-a f)^2 (1+m)}$$

Problem 3127: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{e+f x} \, dx$$

Optimal (type 5, 370 leaves, 6 steps):

$$-\frac{d \left(2 a b c d f^2 m-a^2 d^2 f^2 m-b^2 \left(2 d^2 e^2-4 c d e f+c^2 f^2 (2+m)\right)\right) (a+b x)^{1+m} (c+d x)^{-m}}{2 b^2 (b c-a d) f^3 m}+$$
$$\frac{d^2 (a+b x)^{2+m} (c+d x)^{-m}}{2 b^2 f}+\frac{(d e-c f)^2 (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]}{f^3 m}+$$
$$\Big(d \left(2 a b d f (d e-c f (2-m)) m+a^2 d^2 f^2 (1-m) m-b^2 \left(2 d^2 e^2-2 c d e f (2-m)+c^2 f^2 \left(2-3 m+m^2\right)\right)\right)$$
$$(a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]\Big) \Big/ \left(2 b^2 (b c-a d) f^3 m (1+m)\right)$$

Result (type 5, 319 leaves, 10 steps):

$$-\frac{(d e-c f)^2 (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,m,1+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]}{f^3 m}+$$
$$\frac{d (b c-a d) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]}{b^2 f (1+m)}+$$
$$\frac{(d e-c f)^2 (a+b x)^m (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d (a+b x)}{b c-a d}\right]}{f^3 m}-$$
$$\frac{d (d e-c f) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]}{b f^2 (1+m)}$$

Problem 3128: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{(e+f x)^2} \, dx$$

Optimal (type 5, 316 leaves, 7 steps):

$$-\frac{2 d^2 (d e-c f) (a+b x)^{1+m} (c+d x)^{-m}}{(b c-a d) f^3 m}+\frac{(d e-c f)^2 (a+b x)^{1+m} (c+d x)^{-m}}{f^2 (b e-a f) (e+f x)}+\frac{1}{f^3 (b e-a f) m}$$
$$(d e-c f) (a d f (2-m)-b (2 d e-c f m)) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]+\frac{1}{b (b c-a d) f^3 m (1+m)}$$
$$d^2 (b (2 d e-c f (2-m))-a d f m) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]$$

Result (type 6, 113 leaves, 2 steps):

$$\frac{(b c-a d)^2 (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m,-2+m,2,2+m,-\frac{d (a+b x)}{b c-a d},-\frac{f (a+b x)}{b e-a f}\right]}{b (b e-a f)^2 (1+m)}$$

Problem 3129: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{(e+f x)^3} \, dx$$

Optimal (type 5, 362 leaves, 7 steps):

$$\frac{(b e-a f) (a+b x)^{-1+m} (c+d x)^{2-m}}{2 f^2 (e+f x)^2}+\frac{(a d f (2-m)-b (3 d e-c f (1+m))) (a+b x)^{-1+m} (c+d x)^{2-m}}{2 f^2 (d e-c f) (e+f x)}-$$
$$\left(\left(2 a b d f (2-m) (d e-c f m)-b^2 \left(2 d^2 e^2-2 c d e f m-c^2 f^2 (1-m) m\right)-a^2 d^2 f^2 \left(2-3 m+m^2\right)\right) (a+b x)^{-1+m}\right.$$
$$\left.(c+d x)^{1-m} \text{Hypergeometric2F1}\left[1,-1+m,m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]\right) \Big/ \left(2 f^3 (b e-a f) (d e-c f) (1-m)\right)-$$
$$\frac{d (b c-a d) (a+b x)^{-1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m,-1+m,m,-\frac{d (a+b x)}{b c-a d}\right]}{f^3 (1-m)}$$

Result (type 6, 110 leaves, 2 steps):

$$\frac{(b c-a d)^2 (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m,-2+m,3,2+m,-\frac{d (a+b x)}{b c-a d},-\frac{f (a+b x)}{b e-a f}\right]}{(b e-a f)^3 (1+m)}$$

Problem 3134: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{3-m}}{e+f x} \, dx$$

Optimal (type 5, 488 leaves, 7 steps):

$$\frac{b (b e-a f)^3 (a+b x)^{-3+m} (c+d x)^{4-m}}{(b c-a d) f^4 (3-m)}-\frac{b (b (3 d e-c f (1-m))-a d f (2+m)) (a+b x)^{-2+m} (c+d x)^{4-m}}{6 d^2 f^2}+$$
$$\frac{b (a+b x)^{-1+m} (c+d x)^{4-m}}{3 d f}-\frac{(b e-a f)^3 (a+b x)^{-3+m} (c+d x)^{3-m} \text{Hypergeometric2F1}\left[1,-3+m,-2+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]}{f^4 (3-m)}-$$
$$\frac{1}{6 b^3 d^2 f^4 (2-m) (3-m)} (b c-a d)^2 \left(3 a^2 b d^2 f^2 (d e-c f (3-m)) (1-m) m+a^3 d^3 f^3 m \left(2-3 m+m^2\right)+\right.$$
$$\left.3 a b^2 d f m \left(2 d^2 e^2-2 c d e f (3-m)+c^2 f^2 \left(6-5 m+m^2\right)\right)-b^3 \left(6 d^3 e^3-6 c d^2 e^2 f (3-m)+3 c^2 d e f^2 \left(6-5 m+m^2\right)-c^3 f^3 \left(6-11 m+6 m^2-m^3\right)\right)\right)$$
$$(a+b x)^{-2+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-3+m,-2+m,-1+m,-\frac{d (a+b x)}{b c-a d}\right]$$

Result (type 5, 417 leaves, 13 steps):

$$\frac{(d e-c f)^3 (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,m,1+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]}{f^4 m}+$$
$$\frac{d (b c-a d)^2 (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-2+m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]}{b^3 f (1+m)}-\frac{1}{b^2 f^2 (1+m)}$$
$$d (b c-a d) (d e-c f) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]-$$
$$\frac{(d e-c f)^3 (a+b x)^m (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d (a+b x)}{b c-a d}\right]}{f^4 m}+$$
$$\frac{d (d e-c f)^2 (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]}{b f^3 (1+m)}$$

Problem 3136: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^m (c+d x)^{3-m}}{(e+f x)^3} \, dx$$

Optimal (type 5, 453 leaves, 8 steps):

$$-\frac{3 d^3 (d e-c f) (a+b x)^{1+m} (c+d x)^{-m}}{(b c-a d) f^4 m}-\frac{(d e-c f)^3 (a+b x)^{1+m} (c+d x)^{-m}}{2 f^3 (b e-a f) (e+f x)^2}+$$
$$\frac{(d e-c f)^2 (b (5 d e+c f (1-m))-a d f (6-m)) (a+b x)^{1+m} (c+d x)^{-m}}{2 f^3 (b e-a f)^2 (e+f x)}+\frac{1}{2 f^4 (b e-a f)^2 m}$$
$$(d e-c f) \left(2 a b d f (3-m) (2 d e-c f m)-b^2 \left(6 d^2 e^2-4 c d e f m-c^2 f^2 (1-m) m\right)-a^2 d^2 f^2 \left(6-5 m+m^2\right)\right)$$
$$(a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]+\frac{1}{b (b c-a d) f^4 m (1+m)}$$
$$d^3 (b (3 d e-c f (3-m))-a d f m) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]$$

Result (type 6, 113 leaves, 2 steps):

$$\frac{(b c-a d)^3 (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m,-3+m,3,2+m,-\frac{d (a+b x)}{b c-a d},-\frac{f (a+b x)}{b e-a f}\right]}{b (b e-a f)^3 (1+m)}$$

Problem 3137: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^{1-n} (c+d x)^{1+n}}{b c+a d+2 b d x} \, dx$$

Optimal (type 5, 245 leaves, 6 steps):

$$\frac{(b c-a d) (3-2 n) (a+b x)^{2-n} (c+d x)^{-1+n}}{8 b^3 (1-n)}+\frac{d (a+b x)^{3-n} (c+d x)^{-1+n}}{4 b^3}+$$
$$\frac{(b c-a d)^2 (a+b x)^{1-n} (c+d x)^{-1+n} \text{Hypergeometric2F1}\left[1,-1+n,n,-\frac{b (c+d x)}{d (a+b x)}\right]}{8 b^3 d (1-n)}-$$
$$\frac{(b c-a d)^2 \left(1-2 n^2\right) (a+b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n (c+d x)^n \text{Hypergeometric2F1}\left[-1+n,n,1+n,\frac{b (c+d x)}{b c-a d}\right]}{8 b^2 d^2 (1-n) n}$$

Result (type 5, 319 leaves, 10 steps):

$$-\frac{(b c-a d)^2 (a+b x)^{-n} (c+d x)^n \text{Hypergeometric2F1}\left[1,-n,1-n,-\frac{d (a+b x)}{b (c+d x)}\right]}{8 b^2 d^2 n}+\frac{(b c-a d)^2 (a+b x)^{-n} (c+d x)^n \left(\frac{b (c+d x)}{b c-a d}\right)^{-n} \text{Hypergeometric2F1}\left[-n,-n,1-n,-\frac{d (a+b x)}{b c-a d}\right]}{8 b^2 d^2 n}-\frac{(b c-a d) (a+b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n (c+d x)^{1+n} \text{Hypergeometric2F1}\left[n,1+n,2+n,\frac{b (c+d x)}{b c-a d}\right]}{4 b d^2 (1+n)}+\frac{(a+b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n (c+d x)^{2+n} \text{Hypergeometric2F1}\left[n,2+n,3+n,\frac{b (c+d x)}{b c-a d}\right]}{2 d^2 (2+n)}$$

Problem 3138: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1-n} (c+d x)^{1+n}}{(b c+a d+2 b d x)^2} \, dx$$

Optimal (type 5, 154 leaves, 4 steps):

$$-\frac{(b c-a d) (a+b x)^{1-n} (c+d x)^{-1+n} \text{Hypergeometric2F1}\left[2,1-n,2-n,-\frac{d (a+b x)}{b (c+d x)}\right]}{4 b^3 d (1-n)}+\frac{(a+b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n (c+d x)^{1+n} \text{Hypergeometric2F1}\left[n,1+n,2+n,\frac{b (c+d x)}{b c-a d}\right]}{4 b d^2 (1+n)}$$

Result (type 6, 113 leaves, 2 steps):

$$\frac{(a+b x)^{2-n} (c+d x)^n \left(\frac{b (c+d x)}{b c-a d}\right)^{-n} \text{AppellF1}\left[2-n,-1-n,2,3-n,-\frac{d (a+b x)}{b c-a d},-\frac{2 d (a+b x)}{b c-a d}\right]}{b^2 (b c-a d) (2-n)}$$

Problem 3139: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1-n} (c+d x)^{1+n}}{(b c+a d+2 b d x)^3} \, dx$$

Optimal (type 5, 230 leaves, 7 steps):

$$-\frac{(b c-a d)\,(a+b x)^{1-n}\,(c+d x)^n}{8 b^2 d\,(b c+a d+2 b d x)^2}-\frac{(1+2 n)\,(a+b x)^{1-n}\,(c+d x)^n}{8 b^2 d\,(b c+a d+2 b d x)}-\frac{(1-2 n^2)\,(a+b x)^{-n}\,(c+d x)^n\,\text{Hypergeometric2F1}\left[1,n,1+n,-\frac{b (c+d x)}{d (a+b x)}\right]}{8 b^2 d^2 n}+\frac{(a+b x)^{-n}\left(-\frac{d (a+b x)}{b c-a d}\right)^n (c+d x)^n\,\text{Hypergeometric2F1}\left[n,n,1+n,\frac{b (c+d x)}{b c-a d}\right]}{8 b^2 d^2 n}$$

Result (type 6, 113 leaves, 2 steps):

$$\frac{(a+b x)^{2-n}\,(c+d x)^n\left(\frac{b (c+d x)}{b c-a d}\right)^{-n}\text{AppellF1}\left[2-n,-1-n,3,3-n,-\frac{d (a+b x)}{b c-a d},-\frac{2 d (a+b x)}{b c-a d}\right]}{b^2\,(b c-a d)^2\,(2-n)}$$

Problem 3141: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b x)^m\,(c+d x)^{2-m}}{b c+a d+2 b d x}\,dx$$

Optimal (type 5, 231 leaves, 6 steps):

$$\frac{(b c-a d)\,(1+2 m)\,(a+b x)^{1+m}\,(c+d x)^{-m}}{8 b^3 m}+\frac{d\,(a+b x)^{2+m}\,(c+d x)^{-m}}{4 b^3}+\frac{(b c-a d)^2\,(a+b x)^m\,(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,-\frac{b (c+d x)}{d (a+b x)}\right]}{8 b^3 d m}-\frac{(b c-a d)\,(1-4 m+2 m^2)\,(a+b x)^{1+m}\,(c+d x)^{-m}\left(\frac{b (c+d x)}{b c-a d}\right)^m\text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]}{8 b^3 m\,(1+m)}$$

Result (type 5, 314 leaves, 10 steps):

$$-\frac{(b c-a d)^2\,(a+b x)^m\,(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,m,1+m,-\frac{d (a+b x)}{b (c+d x)}\right]}{8 b^3 d m}+\frac{(b c-a d)\,(a+b x)^{1+m}\,(c+d x)^{-m}\left(\frac{b (c+d x)}{b c-a d}\right)^m\text{Hypergeometric2F1}\left[-1+m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]}{2 b^3\,(1+m)}+\frac{(b c-a d)^2\,(a+b x)^m\,(c+d x)^{-m}\left(\frac{b (c+d x)}{b c-a d}\right)^m\text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d (a+b x)}{b c-a d}\right]}{8 b^3 d m}+\frac{(b c-a d)\,(a+b x)^{1+m}\,(c+d x)^{-m}\left(\frac{b (c+d x)}{b c-a d}\right)^m\text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]}{4 b^3\,(1+m)}$$

Problem 3142: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{(b c+a d+2 b d x)^2} \, dx$$

Optimal (type 5, 144 leaves, 4 steps):

$$-\frac{(b c-a d) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[2, m, 1+m, -\frac{d (a+b x)}{b (c+d x)}\right]}{4 b^3 d m}+\frac{(b c-a d) (a+b x)^m (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m, m, 1+m, -\frac{d (a+b x)}{b c-a d}\right]}{4 b^3 d m}$$

Result (type 6, 93 leaves, 2 steps):

$$\frac{(a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m, -2+m, 2, 2+m, -\frac{d (a+b x)}{b c-a d}, -\frac{2 d (a+b x)}{b c-a d}\right]}{b^3 (1+m)}$$

Problem 3143: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{(b c+a d+2 b d x)^3} \, dx$$

Optimal (type 5, 261 leaves, 7 steps):

$$\frac{(b c-a d) (a+b x)^{-1+m} (c+d x)^{2-m}}{8 b d^2 (b c+a d+2 b d x)^2}+\frac{(1-2 m) (a+b x)^{-1+m} (c+d x)^{2-m}}{8 b d^2 (b c+a d+2 b d x)}-\frac{\left(1-4 m+2 m^2\right) (a+b x)^{-1+m} (c+d x)^{1-m} \text{Hypergeometric2F1}\left[1, -1+m, m, -\frac{d (a+b x)}{b (c+d x)}\right]}{8 b^2 d^2 (1-m)}-\frac{(b c-a d) (a+b x)^{-1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m, -1+m, m, -\frac{d (a+b x)}{b c-a d}\right]}{8 b^3 d^2 (1-m)}$$

Result (type 6, 103 leaves, 2 steps):

$$\frac{(a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m, -2+m, 3, 2+m, -\frac{d (a+b x)}{b c-a d}, -\frac{2 d (a+b x)}{b c-a d}\right]}{b^3 (b c-a d) (1+m)}$$

Test results for the 159 problems in "1.1.1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Problem 111: Unable to integrate problem.

$$\int \frac{1}{(a+b x)^{3/2} (c+d x)^{3/2} \sqrt{e+f x} \sqrt{g+h x}} \, dx$$

Optimal (type 4, 786 leaves, ? steps):

$$-\frac{2 d^3 \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x}}{(b c-a d)^2 (d e-c f) (d g-c h) \sqrt{c+d x}}-\frac{2 b^3 \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{(b c-a d)^2 (b e-a f) (b g-a h) \sqrt{a+b x}}+$$
$$\frac{2 b \left(a^2 d^2 f h-a b d^2 (f g+e h)+b^2 \left(2 d^2 e g+c^2 f h-c d (f g+e h)\right)\right) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{(b c-a d)^2 (b e-a f) (d e-c f) (b g-a h) (d g-c h) \sqrt{a+b x}}-$$
$$\left(2 \sqrt{f g-e h} \left(a^2 d^2 f h-a b d^2 (f g+e h)+b^2 \left(2 d^2 e g+c^2 f h-c d (f g+e h)\right)\right)\right.$$
$$\left.\sqrt{c+d x} \sqrt{-\frac{(b e-a f) (g+h x)}{(f g-e h) (a+b x)}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b g-a h} \sqrt{e+f x}}{\sqrt{f g-e h} \sqrt{a+b x}}\right],-\frac{(b c-a d) (f g-e h)}{(d e-c f) (b g-a h)}\right]\right)\Big/$$
$$\left((b c-a d)^2 (b e-a f) (d e-c f) \sqrt{b g-a h} (d g-c h) \sqrt{\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}} \sqrt{g+h x}\right)-$$
$$\frac{4 b d \sqrt{\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}} \sqrt{g+h x} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b g-a h} \sqrt{e+f x}}{\sqrt{f g-e h} \sqrt{a+b x}}\right],-\frac{(b c-a d) (f g-e h)}{(d e-c f) (b g-a h)}\right]}{(b c-a d)^2 \sqrt{b g-a h} \sqrt{f g-e h} \sqrt{c+d x} \sqrt{-\frac{(b e-a f) (g+h x)}{(f g-e h) (a+b x)}}}$$

Result (type 8, 39 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a+b x)^{3/2} (c+d x)^{3/2} \sqrt{e+f x} \sqrt{g+h x}}, x\right]$$

Problem 137: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (A+B x) (c+d x)^{-m}}{e+f x} \, dx$$

Optimal (type 5, 233 leaves, 5 steps):

$$-\frac{d\,(B e-A f)\,(a+b x)^{1+m}\,(c+d x)^{-m}}{(b c-a d)\,f^2\,m}-\frac{(B e-A f)\,(a+b x)^m\,(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f)\,(c+d x)}{(d e-c f)\,(a+b x)}\right]}{f^2\,m}-\frac{1}{b\,(b c-a d)\,f^2\,m\,(1+m)}$$

$$(a B d f m-b\,(B d e-A d f+B c f m))\,(a+b x)^{1+m}\,(c+d x)^{-m}\left(\frac{b\,(c+d x)}{b c-a d}\right)^m\text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d\,(a+b x)}{b c-a d}\right]$$

Result (type 5, 220 leaves, 7 steps):

$$\frac{(B e-A f)\,(a+b x)^m\,(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,m,1+m,\frac{(d e-c f)\,(a+b x)}{(b e-a f)\,(c+d x)}\right]}{f^2\,m}-$$

$$\frac{(B e-A f)\,(a+b x)^m\,(c+d x)^{-m}\left(\frac{b\,(c+d x)}{b c-a d}\right)^m\text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d\,(a+b x)}{b c-a d}\right]}{f^2\,m}+$$

$$\frac{B\,(a+b x)^{1+m}\,(c+d x)^{-m}\left(\frac{b\,(c+d x)}{b c-a d}\right)^m\text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d\,(a+b x)}{b c-a d}\right]}{b\,f\,(1+m)}$$

Test results for the 34 problems in "1.1.1.5 P(x) (a+b x)^m (c+d x)^n.m"

Test results for the 78 problems in "1.1.1.6 P(x) (a+b x)^m (c+d x)^n (e+f x)^p.m"

Test results for the 35 problems in "1.1.1.7 P(x) (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Test results for the 1071 problems in "1.1.2.2 (c x)^m (a+b x^2)^p.m"

Problem 662: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(\frac{a\,(2+m)\,x^{1+m}}{\sqrt{a+b x^2}}+\frac{b\,(3+m)\,x^{3+m}}{\sqrt{a+b x^2}}\right)dx$$

Optimal (type 3, 17 leaves, ? steps):

$$x^{2+m}\sqrt{a+b x^2}$$

Result (type 5, 127 leaves, 5 steps):

$$\frac{a\,x^{2+m}\sqrt{1+\frac{b\,x^2}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{b\,x^2}{a}\right]}{\sqrt{a+b\,x^2}}+\frac{b\,(3+m)\,x^{4+m}\sqrt{1+\frac{b\,x^2}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{4+m}{2},\frac{6+m}{2},-\frac{b\,x^2}{a}\right]}{(4+m)\sqrt{a+b\,x^2}}$$

Problem 664: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(-\frac{b\,x^{1+m}}{\left(a+b\,x^2\right)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x^2}}\right)dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x^2}}$$

Result (type 5, 123 leaves, 5 steps):

$$\frac{x^m\sqrt{1+\frac{b\,x^2}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{2},\frac{2+m}{2},-\frac{b\,x^2}{a}\right]}{\sqrt{a+b\,x^2}}-\frac{b\,x^{2+m}\sqrt{1+\frac{b\,x^2}{a}}\;\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{b\,x^2}{a}\right]}{a\,(2+m)\sqrt{a+b\,x^2}}$$

Problem 738: Result valid but suboptimal antiderivative.

$$\int (c\,x)^{13/3}\left(a+b\,x^2\right)^{1/3}dx$$

Optimal (type 3, 195 leaves, 6 steps):

$$-\frac{5\,a^2\,c^3\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{108\,b^2}+\frac{a\,c\,(c\,x)^{10/3}\left(a+b\,x^2\right)^{1/3}}{36\,b}+\frac{(c\,x)^{16/3}\left(a+b\,x^2\right)^{1/3}}{6\,c}-\frac{5\,a^3\,c^{13/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{54\,\sqrt{3}\,b^{8/3}}-\frac{5\,a^3\,c^{13/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\left(a+b\,x^2\right)^{1/3}\right]}{108\,b^{8/3}}$$

Result (type 3, 275 leaves, 12 steps):

$$-\frac{5 a^2 c^3 (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{108 b^2}+\frac{a c (c x)^{10/3} \left(a+b x^2\right)^{1/3}}{36 b}+\frac{(c x)^{16/3} \left(a+b x^2\right)^{1/3}}{6 c}-$$

$$\frac{5 a^3 c^{13/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3}\, c^{2/3}}\right]}{54 \sqrt{3}\, b^{8/3}}-\frac{5 a^3 c^{13/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{162 b^{8/3}}+\frac{5 a^3 c^{13/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{324 b^{8/3}}$$

Problem 739: Result valid but suboptimal antiderivative.

$$\int (c x)^{7/3} \left(a+b x^2\right)^{1/3} \, dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$\frac{a c (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{12 b}+\frac{(c x)^{10/3} \left(a+b x^2\right)^{1/3}}{4 c}+\frac{a^2 c^{7/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{6 \sqrt{3}\, b^{5/3}}+\frac{a^2 c^{7/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{12 b^{5/3}}$$

Result (type 3, 244 leaves, 11 steps):

$$\frac{a c (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{12 b}+\frac{(c x)^{10/3} \left(a+b x^2\right)^{1/3}}{4 c}+\frac{a^2 c^{7/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3}\, c^{2/3}}\right]}{6 \sqrt{3}\, b^{5/3}}+$$

$$\frac{a^2 c^{7/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{18 b^{5/3}}-\frac{a^2 c^{7/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{36 b^{5/3}}$$

Problem 740: Result valid but suboptimal antiderivative.

$$\int (c x)^{1/3} \left(a+b x^2\right)^{1/3} \, dx$$

Optimal (type 3, 133 leaves, 4 steps):

$$\frac{(c x)^{4/3} \left(a+b x^2\right)^{1/3}}{2 c}-\frac{a c^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2 \sqrt{3}\, b^{2/3}}-\frac{a c^{1/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{4 b^{2/3}}$$

Result (type 3, 211 leaves, 10 steps):

$$\frac{(c x)^{4/3} \left(a+b x^2\right)^{1/3}}{2 c}-\frac{a c^{1/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3}\, c^{2/3}}\right]}{2 \sqrt{3}\, b^{2/3}}-\frac{a c^{1/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{6 b^{2/3}}+\frac{a c^{1/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{12 b^{2/3}}$$

Problem 741: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^2\right)^{1/3}}{(c x)^{5/3}} \, dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$-\frac{3 \left(a+b x^2\right)^{1/3}}{2 c (c x)^{2/3}}-\frac{\sqrt{3}\, b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2 c^{5/3}}-\frac{3 b^{1/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{4 c^{5/3}}$$

Result (type 3, 208 leaves, 10 steps):

$$-\frac{3 \left(a+b x^2\right)^{1/3}}{2 c (c x)^{2/3}}-\frac{\sqrt{3}\, b^{1/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3}\, c^{2/3}}\right]}{2 c^{5/3}}-\frac{b^{1/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{2 c^{5/3}}+\frac{b^{1/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{4 c^{5/3}}$$

Problem 754: Result valid but suboptimal antiderivative.

$$\int (c x)^{13/3} \left(a+b x^2\right)^{4/3} \, dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$-\frac{5 a^3 c^3 (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{324 b^2}+\frac{a^2 c (c x)^{10/3} \left(a+b x^2\right)^{1/3}}{108 b}+\frac{a (c x)^{16/3} \left(a+b x^2\right)^{1/3}}{18 c}+$$
$$\frac{(c x)^{16/3} \left(a+b x^2\right)^{4/3}}{8 c}-\frac{5 a^4 c^{13/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{162 \sqrt{3}\, b^{8/3}}-\frac{5 a^4 c^{13/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{324 b^{8/3}}$$

Result (type 3, 303 leaves, 13 steps):

$$-\frac{5 a^3 c^3 (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{324 b^2}+\frac{a^2 c (c x)^{10/3} \left(a+b x^2\right)^{1/3}}{108 b}+\frac{a (c x)^{16/3} \left(a+b x^2\right)^{1/3}}{18 c}+\frac{(c x)^{16/3} \left(a+b x^2\right)^{4/3}}{8 c}-$$
$$\frac{5 a^4 c^{13/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3}\, c^{2/3}}\right]}{162 \sqrt{3}\, b^{8/3}}-\frac{5 a^4 c^{13/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{486 b^{8/3}}+\frac{5 a^4 c^{13/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{972 b^{8/3}}$$

Problem 755: Result valid but suboptimal antiderivative.

$$\int (c\,x)^{7/3}\left(a+b\,x^2\right)^{4/3}\,dx$$

Optimal (type 3, 192 leaves, 6 steps):

$$\frac{a^2\,c\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{27\,b}+\frac{a\,(c\,x)^{10/3}\left(a+b\,x^2\right)^{1/3}}{9\,c}+\frac{(c\,x)^{10/3}\left(a+b\,x^2\right)^{4/3}}{6\,c}+\frac{2\,a^3\,c^{7/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{27\,\sqrt{3}\,b^{5/3}}+\frac{a^3\,c^{7/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\left(a+b\,x^2\right)^{1/3}\right]}{27\,b^{5/3}}$$

Result (type 3, 272 leaves, 12 steps):

$$\frac{a^2\,c\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{27\,b}+\frac{a\,(c\,x)^{10/3}\left(a+b\,x^2\right)^{1/3}}{9\,c}+\frac{(c\,x)^{10/3}\left(a+b\,x^2\right)^{4/3}}{6\,c}+\frac{2\,a^3\,c^{7/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{27\,\sqrt{3}\,b^{5/3}}+\frac{2\,a^3\,c^{7/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{81\,b^{5/3}}-\frac{a^3\,c^{7/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{\left(a+b\,x^2\right)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{81\,b^{5/3}}$$

Problem 756: Result valid but suboptimal antiderivative.

$$\int (c\,x)^{1/3}\left(a+b\,x^2\right)^{4/3}\,dx$$

Optimal (type 3, 163 leaves, 5 steps):

$$\frac{a\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{3\,c}+\frac{(c\,x)^{4/3}\left(a+b\,x^2\right)^{4/3}}{4\,c}-\frac{a^2\,c^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{2/3}}-\frac{a^2\,c^{1/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\left(a+b\,x^2\right)^{1/3}\right]}{6\,b^{2/3}}$$

Result (type 3, 243 leaves, 11 steps):

$$\frac{a\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{3\,c}+\frac{(c\,x)^{4/3}\left(a+b\,x^2\right)^{4/3}}{4\,c}-\frac{a^2\,c^{1/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{3\,\sqrt{3}\,b^{2/3}}-\frac{a^2\,c^{1/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{9\,b^{2/3}}+\frac{a^2\,c^{1/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{\left(a+b\,x^2\right)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{18\,b^{2/3}}$$

Problem 757: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^2\right)^{4/3}}{(c x)^{5/3}}\,dx$$

Optimal (type 3, 153 leaves, 5 steps):

$$\frac{2 b (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{c^3}-\frac{3 \left(a+b x^2\right)^{4/3}}{2 c (c x)^{2/3}}-\frac{2 a b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{5/3}}-\frac{a b^{1/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{c^{5/3}}$$

Result (type 3, 233 leaves, 11 steps):

$$\frac{2 b (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{c^3}-\frac{3 \left(a+b x^2\right)^{4/3}}{2 c (c x)^{2/3}}-\frac{2 a b^{1/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{\sqrt{3} c^{5/3}}-\frac{2 a b^{1/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{3 c^{5/3}}+\frac{a b^{1/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{3 c^{5/3}}$$

Problem 758: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^2\right)^{4/3}}{(c x)^{11/3}}\,dx$$

Optimal (type 3, 157 leaves, 5 steps):

$$-\frac{3 b \left(a+b x^2\right)^{1/3}}{2 c^3 (c x)^{2/3}}-\frac{3 \left(a+b x^2\right)^{4/3}}{8 c (c x)^{8/3}}-\frac{\sqrt{3} b^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2 c^{11/3}}-\frac{3 b^{4/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{4 c^{11/3}}$$

Result (type 3, 234 leaves, 11 steps):

$$-\frac{3 b \left(a+b x^2\right)^{1/3}}{2 c^3 (c x)^{2/3}}-\frac{3 \left(a+b x^2\right)^{4/3}}{8 c (c x)^{8/3}}-\frac{\sqrt{3} b^{4/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{2 c^{11/3}}-\frac{b^{4/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{2 c^{11/3}}+\frac{b^{4/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{4 c^{11/3}}$$

Problem 771: Result valid but suboptimal antiderivative.

$$\int \frac{(c x)^{19/3}}{\left(a+b x^2\right)^{2/3}}\,dx$$

Optimal (type 3, 198 leaves, 6 steps):

$$\frac{10 a^2 c^5 (c x)^{4/3} (a+b x^2)^{1/3}}{27 b^3}-\frac{2 a c^3 (c x)^{10/3} (a+b x^2)^{1/3}}{9 b^2}+\frac{c (c x)^{16/3} (a+b x^2)^{1/3}}{6 b}+$$
$$\frac{20 a^3 c^{19/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a+b x^2)^{1/3}}}{\sqrt{3}}\right]}{27 \sqrt{3} b^{11/3}}+\frac{10 a^3 c^{19/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} (a+b x^2)^{1/3}\right]}{27 b^{11/3}}$$

Result (type 3, 278 leaves, 12 steps):

$$\frac{10 a^2 c^5 (c x)^{4/3} (a+b x^2)^{1/3}}{27 b^3}-\frac{2 a c^3 (c x)^{10/3} (a+b x^2)^{1/3}}{9 b^2}+\frac{c (c x)^{16/3} (a+b x^2)^{1/3}}{6 b}+$$
$$\frac{20 a^3 c^{19/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{27 \sqrt{3} b^{11/3}}+\frac{20 a^3 c^{19/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}\right]}{81 b^{11/3}}-\frac{10 a^3 c^{19/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{(a+b x^2)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}\right]}{81 b^{11/3}}$$

Problem 772: Result valid but suboptimal antiderivative.

$$\int \frac{(c x)^{13/3}}{(a+b x^2)^{2/3}} \, dx$$

Optimal (type 3, 167 leaves, 5 steps):

$$-\frac{5 a c^3 (c x)^{4/3} (a+b x^2)^{1/3}}{12 b^2}+\frac{c (c x)^{10/3} (a+b x^2)^{1/3}}{4 b}-\frac{5 a^2 c^{13/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a+b x^2)^{1/3}}}{\sqrt{3}}\right]}{6 \sqrt{3} b^{8/3}}-\frac{5 a^2 c^{13/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} (a+b x^2)^{1/3}\right]}{12 b^{8/3}}$$

Result (type 3, 247 leaves, 11 steps):

$$-\frac{5 a c^3 (c x)^{4/3} (a+b x^2)^{1/3}}{12 b^2}+\frac{c (c x)^{10/3} (a+b x^2)^{1/3}}{4 b}-\frac{5 a^2 c^{13/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{6 \sqrt{3} b^{8/3}}-$$
$$\frac{5 a^2 c^{13/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}\right]}{18 b^{8/3}}+\frac{5 a^2 c^{13/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{(a+b x^2)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}\right]}{36 b^{8/3}}$$

Problem 773: Result valid but suboptimal antiderivative.

$$\int \frac{(c x)^{7/3}}{(a+b x^2)^{2/3}} \, dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$\frac{c\,(c\,x)^{4/3}\,(a+b\,x^2)^{1/3}}{2\,b}+\frac{a\,c^{7/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,(a+b\,x^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{5/3}}+\frac{a\,c^{7/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\,(a+b\,x^2)^{1/3}\right]}{2\,b^{5/3}}$$

Result (type 3, 209 leaves, 10 steps):

$$\frac{c\,(c\,x)^{4/3}\,(a+b\,x^2)^{1/3}}{2\,b}+\frac{a\,c^{7/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{(a+b\,x^2)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{\sqrt{3}\,b^{5/3}}+\frac{a\,c^{7/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{(a+b\,x^2)^{1/3}}\right]}{3\,b^{5/3}}-\frac{a\,c^{7/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{(a+b\,x^2)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{(a+b\,x^2)^{1/3}}\right]}{6\,b^{5/3}}$$

Problem 774: Result valid but suboptimal antiderivative.

$$\int\frac{(c\,x)^{1/3}}{(a+b\,x^2)^{2/3}}\,dx$$

Optimal (type 3, 106 leaves, 3 steps):

$$-\frac{\sqrt{3}\,c^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,(a+b\,x^2)^{1/3}}}{\sqrt{3}}\right]}{2\,b^{2/3}}-\frac{3\,c^{1/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\,(a+b\,x^2)^{1/3}\right]}{4\,b^{2/3}}$$

Result (type 3, 183 leaves, 9 steps):

$$-\frac{\sqrt{3}\,c^{1/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{(a+b\,x^2)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{2\,b^{2/3}}-\frac{c^{1/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{(a+b\,x^2)^{1/3}}\right]}{2\,b^{2/3}}+\frac{c^{1/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{(a+b\,x^2)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{(a+b\,x^2)^{1/3}}\right]}{4\,b^{2/3}}$$

Test results for the 349 problems in "1.1.2.3 (a+b x^2)^p (c+d x^2)^q.m"

Problem 301: Result unnecessarily involves higher level functions.

$$\int\frac{(1-2\,x^2)^m}{\sqrt{1-x^2}}\,dx$$

Optimal (type 5, 62 leaves, ? steps):

$$-\frac{2^{-2-m}\,\sqrt{x^2}\,(2-4\,x^2)^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},(1-2\,x^2)^2\right]}{(1+m)\,x}$$

Result (type 6, 23 leaves, 1 step):

$$x \,\text{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 2 x^2, x^2\right]$$

Test results for the 1156 problems in "1.1.2.4 (e x)^m (a+b x^2)^p (c+d x^2)^q.m"

Test results for the 115 problems in "1.1.2.5 (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 51 problems in "1.1.2.6 (g x)^m (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 174 problems in "1.1.2.8 P(x) (c x)^m (a+b x^2)^p.m"

Test results for the 3078 problems in "1.1.3.2 (c x)^m (a+b x^n)^p.m"

Problem 516: Result valid but suboptimal antiderivative.

$$\int x^4 \left(a + b x^3\right)^{1/3} \, dx$$

Optimal (type 3, 120 leaves, 3 steps):

$$\frac{a x^2 \left(a + b x^3\right)^{1/3}}{18 b} + \frac{1}{6} x^5 \left(a + b x^3\right)^{1/3} + \frac{a^2 \,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{5/3}} + \frac{a^2 \,\text{Log}\left[b^{1/3} x - \left(a + b x^3\right)^{1/3}\right]}{18 b^{5/3}}$$

Result (type 3, 173 leaves, 9 steps):

$$\frac{a x^2 \left(a + b x^3\right)^{1/3}}{18 b} + \frac{1}{6} x^5 \left(a + b x^3\right)^{1/3} + \frac{a^2 \,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{5/3}} + \frac{a^2 \,\text{Log}\left[1 - \frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{27 b^{5/3}} - \frac{a^2 \,\text{Log}\left[1 + \frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}} + \frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{54 b^{5/3}}$$

Problem 517: Result valid but suboptimal antiderivative.

$$\int x \left(a + b x^3\right)^{1/3} \, dx$$

Optimal (type 3, 94 leaves, 2 steps):

$$\frac{1}{3} x^2 \left(a+b x^3\right)^{1/3}-\frac{a \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}}-\frac{a \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 b^{2/3}}$$

Result (type 3, 145 leaves, 8 steps) :

$$\frac{1}{3} x^2 \left(a+b x^3\right)^{1/3}-\frac{a \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}}-\frac{a \text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{2/3}}+\frac{a \text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{18 b^{2/3}}$$

Problem 518: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^2} \, dx$$

Optimal (type 3, 88 leaves, 2 steps) :

$$-\frac{\left(a+b x^3\right)^{1/3}}{x}-\frac{b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{2} b^{1/3} \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]$$

Result (type 3, 138 leaves, 8 steps) :

$$-\frac{\left(a+b x^3\right)^{1/3}}{x}-\frac{b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{3} b^{1/3} \text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]+\frac{1}{6} b^{1/3} \text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]$$

Problem 567: Result valid but suboptimal antiderivative.

$$\int \frac{x^7}{\left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 123 leaves, 3 steps) :

$$-\frac{5 a x^2 \left(a+b x^3\right)^{1/3}}{18 b^2}+\frac{x^5 \left(a+b x^3\right)^{1/3}}{6 b}-\frac{5 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3}}-\frac{5 a^2 \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{18 b^{8/3}}$$

Result (type 3, 176 leaves, 9 steps) :

$$-\frac{5\,a\,x^2\,\left(a+b\,x^3\right)^{1/3}}{18\,b^2}+\frac{x^5\,\left(a+b\,x^3\right)^{1/3}}{6\,b}-\frac{5\,a^2\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,b^{8/3}}-\frac{5\,a^2\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{27\,b^{8/3}}+\frac{5\,a^2\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{54\,b^{8/3}}$$

Problem 568: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 97 leaves, 2 steps):

$$\frac{x^2\,\left(a+b\,x^3\right)^{1/3}}{3\,b}+\frac{2\,a\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{5/3}}+\frac{a\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{3\,b^{5/3}}$$

Result (type 3, 148 leaves, 8 steps):

$$\frac{x^2\,\left(a+b\,x^3\right)^{1/3}}{3\,b}+\frac{2\,a\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{5/3}}+\frac{2\,a\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{5/3}}-\frac{a\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{5/3}}$$

Problem 569: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 72 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}-\frac{\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,b^{2/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}-\frac{\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}+\frac{\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\,b^{2/3}}$$

Problem 581: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(1-x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 53 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{2} \text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]$$

Result (type 3, 87 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{6} \text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{1}{3} \text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]$$

Problem 2271: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\left(a+b x^{3/2}\right)^{2/3}} \, dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$-\frac{5 a x \left(a+b x^{3/2}\right)^{1/3}}{9 b^2}+\frac{x^{5/2} \left(a+b x^{3/2}\right)^{1/3}}{3 b}-\frac{10 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} \sqrt{x}}{\left(a+b x^{3/2}\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3}}-\frac{5 a^2 \text{Log}\left[b^{1/3} \sqrt{x}-\left(a+b x^{3/2}\right)^{1/3}\right]}{9 b^{8/3}}$$

Result (type 3, 198 leaves, 10 steps):

$$-\frac{5 a x \left(a+b x^{3/2}\right)^{1/3}}{9 b^2}+\frac{x^{5/2} \left(a+b x^{3/2}\right)^{1/3}}{3 b}-\frac{10 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} \sqrt{x}}{\left(a+b x^{3/2}\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3}}-\frac{10 a^2 \text{Log}\left[1-\frac{b^{1/3} \sqrt{x}}{\left(a+b x^{3/2}\right)^{1/3}}\right]}{27 b^{8/3}}+\frac{5 a^2 \text{Log}\left[1+\frac{b^{2/3} x}{\left(a+b x^{3/2}\right)^{2/3}}+\frac{b^{1/3} \sqrt{x}}{\left(a+b x^{3/2}\right)^{1/3}}\right]}{27 b^{8/3}}$$

Problem 2272: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b x^{3/2}\right)^{2/3}} \, dx$$

Optimal (type 3, 82 leaves, 2 steps):

$$-\frac{2\,\mathrm{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\sqrt{x}}{\left(a+b\,x^{3/2}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}-\frac{\mathrm{Log}\left[b^{1/3}\sqrt{x}-\left(a+b\,x^{3/2}\right)^{1/3}\right]}{b^{2/3}}$$

Result (type 3, 140 leaves, 8 steps):

$$-\frac{2\,\mathrm{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\sqrt{x}}{\left(a+b\,x^{3/2}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}-\frac{2\,\mathrm{Log}\left[1-\frac{b^{1/3}\sqrt{x}}{\left(a+b\,x^{3/2}\right)^{1/3}}\right]}{3\,b^{2/3}}+\frac{\mathrm{Log}\left[1+\frac{b^{2/3}\,x}{\left(a+b\,x^{3/2}\right)^{2/3}}+\frac{b^{1/3}\sqrt{x}}{\left(a+b\,x^{3/2}\right)^{1/3}}\right]}{3\,b^{2/3}}$$

Problem 2686: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(-\frac{b\,n\,x^{-1+m+n}}{2\,\left(a+b\,x^n\right)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x^n}}\right)dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m\sqrt{1+\frac{b\,x^n}{a}}\;\mathrm{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{n},\frac{m+n}{n},-\frac{b\,x^n}{a}\right]}{\sqrt{a+b\,x^n}}-\frac{b\,n\,x^{m+n}\sqrt{1+\frac{b\,x^n}{a}}\;\mathrm{Hypergeometric2F1}\left[\frac{3}{2},\frac{m+n}{n},2+\frac{m}{n},-\frac{b\,x^n}{a}\right]}{2\,a\,(m+n)\,\sqrt{a+b\,x^n}}$$

Problem 2697: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(\frac{6\,a\,x^2}{b\,(4+m)\,\sqrt{a+b\,x^{-2+m}}}+\frac{x^m}{\sqrt{a+b\,x^{-2+m}}}\right)dx$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2\,x^3\sqrt{a+b\,x^{-2+m}}}{b\,(4+m)}$$

Result (type 5, 160 leaves, 5 steps):

$$\frac{2\,a\,x^3\,\sqrt{1+\frac{b\,x^{-2+m}}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{3}{2-m},-\frac{1+m}{2-m},-\frac{b\,x^{-2+m}}{a}\right]}{b\,(4+m)\,\sqrt{a+b\,x^{-2+m}}}+\frac{x^{1+m}\,\sqrt{1+\frac{b\,x^{-2+m}}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{1+m}{2-m},\frac{1-2\,m}{2-m},-\frac{b\,x^{-2+m}}{a}\right]}{(1+m)\,\sqrt{a+b\,x^{-2+m}}}$$

Problem 2699: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(-\frac{b\,n\,x^{-1+m+n}}{2\,\left(a+b\,x^n\right)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x^n}}\right)dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m\,\sqrt{1+\frac{b\,x^n}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{n},\frac{m+n}{n},-\frac{b\,x^n}{a}\right]}{\sqrt{a+b\,x^n}}-\frac{b\,n\,x^{m+n}\,\sqrt{1+\frac{b\,x^n}{a}}\;\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{m+n}{n},2+\frac{m}{n},-\frac{b\,x^n}{a}\right]}{2\,a\,(m+n)\,\sqrt{a+b\,x^n}}$$

Test results for the 385 problems in "1.1.3.3 (a+b x^n)^p (c+d x^n)^q.m"

Problem 34: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{7/3}}{a-b\,x^3}\,dx$$

Optimal (type 5, 483 leaves, 22 steps):

$$-\frac{7}{5} a x \left(a+b x^3\right)^{1/3}-\frac{1}{5} x \left(a+b x^3\right)^{4/3}-\frac{4\times 2^{1/3} a^{5/3} \text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3}}-\frac{2\times 2^{1/3} a^{5/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3}}-$$
$$\frac{7 a^2 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{5 \left(a+b x^3\right)^{2/3}}-\frac{2\times 2^{1/3} a^{5/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}+$$
$$\frac{2\times 2^{1/3} a^{5/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}-\frac{4\times 2^{1/3} a^{5/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}+\frac{2^{1/3} a^{5/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}$$

Result (type 6, 56 leaves, 2 steps):

$$\frac{a x \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},1,-\frac{7}{3},\frac{4}{3},\frac{b x^3}{a},-\frac{b x^3}{a}\right]}{\left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 35: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{a-b x^3}\, dx$$

Optimal (type 5, 464 leaves, 21 steps):

$$-\frac{1}{2} x \left(a+b x^3\right)^{1/3}-\frac{2\times 2^{1/3} a^{2/3} \text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3}}-\frac{2^{1/3} a^{2/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3}}-$$
$$\frac{a x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 \left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} a^{2/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}+$$
$$\frac{2^{1/3} a^{2/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}-\frac{2\times 2^{1/3} a^{2/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3}}+\frac{a^{2/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} b^{1/3}}$$

Result (type 6, 55 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},1,-\frac{4}{3},\frac{4}{3},\frac{b x^3}{a},-\frac{b x^3}{a}\right]}{\left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{a-b x^3} \, dx$$

Optimal (type 3, 398 leaves, 14 steps):

$$-\frac{2^{1/3} \operatorname{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{1/3} b^{1/3}}-\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, a^{1/3} b^{1/3}}-\frac{\operatorname{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{1/3} b^{1/3}}+$$
$$\frac{\operatorname{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{1/3} b^{1/3}}-\frac{2^{1/3} \operatorname{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\, a^{1/3} b^{1/3}}+\frac{\operatorname{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{1/3} b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3},1,-\frac{1}{3},\frac{4}{3},\frac{b x^3}{a},-\frac{b x^3}{a}\right]}{a \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 37: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^3\right) \left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 5, 452 leaves, 17 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, a^{4/3} b^{1/3}}-\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3} \sqrt{3}\, a^{4/3} b^{1/3}}+\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2\, a \left(a+b x^3\right)^{2/3}}-$$
$$\frac{\operatorname{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{4/3} b^{1/3}}+\frac{\operatorname{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{4/3} b^{1/3}}-\frac{\operatorname{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{4/3} b^{1/3}}+\frac{\operatorname{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3} a^{4/3} b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3},1,\frac{2}{3},\frac{4}{3},\frac{b x^3}{a},-\frac{b x^3}{a}\right]}{a \left(a+b x^3\right)^{2/3}}$$

Problem 38: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^3\right) \left(a+b x^3\right)^{5/3}} \, dx$$

Optimal (type 5, 473 leaves, 21 steps):

$$\frac{x}{4 a^2 \left(a+b x^3\right)^{2/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3} \sqrt{3}\, a^{7/3} b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{2/3} \sqrt{3}\, a^{7/3} b^{1/3}}+\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 a^2 \left(a+b x^3\right)^{2/3}}-$$

$$\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3} a^{7/3} b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3} a^{7/3} b^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{7/3} b^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{24\times 2^{2/3} a^{7/3} b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},1,\frac{5}{3},\frac{4}{3},\frac{b x^3}{a},-\frac{b x^3}{a}\right]}{a^2 \left(a+b x^3\right)^{2/3}}$$

Problem 39: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^3\right) \left(a+b x^3\right)^{8/3}} \, dx$$

Optimal (type 5, 492 leaves, 22 steps):

$$\frac{x}{10 a^2 \left(a+b x^3\right)^{5/3}}+\frac{13 x}{40 a^3 \left(a+b x^3\right)^{2/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{2/3} \sqrt{3}\, a^{10/3} b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{8\times 2^{2/3} \sqrt{3}\, a^{10/3} b^{1/3}}+\frac{9 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{20 a^3 \left(a+b x^3\right)^{2/3}}-$$

$$\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{24\times 2^{2/3} a^{10/3} b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{24\times 2^{2/3} a^{10/3} b^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3} a^{10/3} b^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{48\times 2^{2/3} a^{10/3} b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},1,\frac{8}{3},\frac{4}{3},\frac{b x^3}{a},-\frac{b x^3}{a}\right]}{a^3 \left(a+b x^3\right)^{2/3}}$$

Problem 86: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{8/3}}{c+d x^3} \, dx$$

Optimal (type 3, 331 leaves, 5 steps):

$$-\frac{b\,(6 b c-11 a d)\,x\,\left(a+b x^3\right)^{2/3}}{18 d^2}+\frac{b x\,\left(a+b x^3\right)^{5/3}}{6 d}+\frac{b^{2/3}\left(9 b^2 c^2-24 a b c d+20 a^2 d^2\right)\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, d^3}-\frac{(b c-a d)^{8/3}\,\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} d^3}-\frac{(b c-a d)^{8/3}\,\text{Log}\left[c+d x^3\right]}{6 c^{2/3} d^3}+\frac{(b c-a d)^{8/3}\,\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} d^3}-\frac{b^{2/3}\left(9 b^2 c^2-24 a b c d+20 a^2 d^2\right)\text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{18 d^3}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^2 x\,\left(a+b x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{8}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c\,\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 87: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{5/3}}{c+d x^3} \, dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$\frac{b x\,\left(a+b x^3\right)^{2/3}}{3 d}-\frac{b^{2/3}\,(3 b c-5 a d)\,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, d^2}+\frac{(b c-a d)^{5/3}\,\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} d^2}+\frac{(b c-a d)^{5/3}\,\text{Log}\left[c+d x^3\right]}{6 c^{2/3} d^2}-\frac{(b c-a d)^{5/3}\,\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} d^2}+\frac{b^{2/3}\,(3 b c-5 a d)\,\text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 d^2}$$

Result (type 6, 60 leaves, 2 steps):

$$\frac{a x\,\left(a+b x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{5}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c\,\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 88: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{b^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d}-\frac{(b c-a d)^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} d}-$$
$$\frac{(b c-a d)^{2/3} \operatorname{Log}\left[c+d x^3\right]}{6 c^{2/3} d}+\frac{(b c-a d)^{2/3} \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} d}-\frac{b^{2/3} \operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 d}$$

Result (type 6, 59 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 89: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 148 leaves, 1 step):

$$\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} (b c-a d)^{1/3}}+\frac{\operatorname{Log}\left[c+d x^3\right]}{6 c^{2/3} (b c-a d)^{1/3}}-\frac{\operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} (b c-a d)^{1/3}}$$

Result (type 3, 207 leaves, 7 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{2/3} (b c-a d)^{1/3}}-\frac{\operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{2/3} (b c-a d)^{1/3}}+\frac{\operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{2/3} (b c-a d)^{1/3}}$$

Problem 90: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 179 leaves, 2 steps):

$$\frac{b x}{a (b c-a d) \left(a+b x^3\right)^{1/3}}-\frac{d \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} (b c-a d)^{4/3}}-\frac{d \text{Log}\left[c+d x^3\right]}{6 c^{2/3} (b c-a d)^{4/3}}+\frac{d \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} (b c-a d)^{4/3}}$$

Result (type 3, 238 leaves, 8 steps):

$$\frac{b x}{a (b c-a d) \left(a+b x^3\right)^{1/3}}-\frac{d \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{2/3} (b c-a d)^{4/3}}+\frac{d \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{2/3} (b c-a d)^{4/3}}-\frac{d \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{2/3} (b c-a d)^{4/3}}$$

Problem 91: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{7/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 226 leaves, 4 steps):

$$\frac{b x}{4 a (b c-a d) \left(a+b x^3\right)^{4/3}}+\frac{b (3 b c-7 a d) x}{4 a^2 (b c-a d)^2 \left(a+b x^3\right)^{1/3}}+\frac{d^2 \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} (b c-a d)^{7/3}}+\frac{d^2 \text{Log}\left[c+d x^3\right]}{6 c^{2/3} (b c-a d)^{7/3}}-\frac{d^2 \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} (b c-a d)^{7/3}}$$

Result (type 5, 621 leaves, 2 steps):

$$-\frac{1}{40 c^4 (b c-a d)^2 x^5 \left(a+b x^3\right)^{10/3}}\Bigg(70 c^4 (b c-a d) x^3 \left(a+b x^3\right)^2+105 c^3 d (b c-a d) x^6 \left(a+b x^3\right)^2+$$
$$45 c^2 d^2 (b c-a d) x^9 \left(a+b x^3\right)^2+280 c^5 \left(a+b x^3\right)^3+420 c^4 d x^3 \left(a+b x^3\right)^3+180 c^3 d^2 x^6 \left(a+b x^3\right)^3-$$
$$280 c^5 \left(a+b x^3\right)^3 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-420 c^4 d x^3 \left(a+b x^3\right)^3 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-$$
$$180 c^3 d^2 x^6 \left(a+b x^3\right)^3 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-33 c^2 (b c-a d)^3 x^9 \text{Hypergeometric2F1}\left[2,\frac{10}{3},\frac{13}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-$$
$$60 c d (b c-a d)^3 x^{12} \text{Hypergeometric2F1}\left[2,\frac{10}{3},\frac{13}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-27 d^2 (b c-a d)^3 x^{15} \text{Hypergeometric2F1}\left[2,\frac{10}{3},\frac{13}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-$$
$$9 c^2 (b c-a d)^3 x^9 \text{HypergeometricPFQ}\left[\left\{2,2,\frac{10}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-18 c d (b c-a d)^3 x^{12}$$
$$\text{HypergeometricPFQ}\left[\left\{2,2,\frac{10}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-9 d^2 (b c-a d)^3 x^{15} \text{HypergeometricPFQ}\left[\left\{2,2,\frac{10}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]\Bigg)$$

Problem 92: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{10/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 280 leaves, 5 steps):

$$\frac{b x}{7 a (b c-a d) \left(a+b x^3\right)^{7/3}}+\frac{b (6 b c-13 a d) x}{28 a^2 (b c-a d)^2 \left(a+b x^3\right)^{4/3}}+\frac{b \left(18 b^2 c^2-57 a b c d+67 a^2 d^2\right) x}{28 a^3 (b c-a d)^3 \left(a+b x^3\right)^{1/3}}-$$

$$\frac{d^3 \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} (b c-a d)^{10/3}}-\frac{d^3 \operatorname{Log}\left[c+d x^3\right]}{6 c^{2/3} (b c-a d)^{10/3}}+\frac{d^3 \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} (b c-a d)^{10/3}}$$

Result (type 5, 1172 leaves, 2 steps):

$$-\frac{1}{5096\,c^5\,(b\,c-a\,d)^3\,x^8\,(a+b\,x^3)^{13/3}}\Bigg(7280\,c^5\,(b\,c-a\,d)^2\,x^6\,(a+b\,x^3)^2+16\,380\,c^4\,d\,(b\,c-a\,d)^2\,x^9\,(a+b\,x^3)^2+14\,040\,c^3\,d^2\,(b\,c-a\,d)^2\,x^{12}\,(a+b\,x^3)^2+$$
$$4212\,c^2\,d^3\,(b\,c-a\,d)^2\,x^{15}\,(a+b\,x^3)^2+12\,740\,c^6\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)^3+28\,665\,c^5\,d\,(b\,c-a\,d)\,x^6\,(a+b\,x^3)^3+$$
$$24\,570\,c^4\,d^2\,(b\,c-a\,d)\,x^9\,(a+b\,x^3)^3+7371\,c^3\,d^3\,(b\,c-a\,d)\,x^{12}\,(a+b\,x^3)^3+50\,960\,c^7\,(a+b\,x^3)^4+114\,660\,c^6\,d\,x^3\,(a+b\,x^3)^4+$$
$$98\,280\,c^5\,d^2\,x^6\,(a+b\,x^3)^4+29\,484\,c^4\,d^3\,x^9\,(a+b\,x^3)^4-50\,960\,c^7\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$114\,660\,c^6\,d\,x^3\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-98\,280\,c^5\,d^2\,x^6\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$29\,484\,c^4\,d^3\,x^9\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-5796\,c^3\,(b\,c-a\,d)^4\,x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{13}{3},\frac{16}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$15\,246\,c^2\,d\,(b\,c-a\,d)^4\,x^{15}\,\text{Hypergeometric2F1}\left[2,\frac{13}{3},\frac{16}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$13\,608\,c\,d^2\,(b\,c-a\,d)^4\,x^{18}\,\text{Hypergeometric2F1}\left[2,\frac{13}{3},\frac{16}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-4158\,d^3\,(b\,c-a\,d)^4\,x^{21}$$
$$\text{Hypergeometric2F1}\left[2,\frac{13}{3},\frac{16}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-2646\,c^3\,(b\,c-a\,d)^4\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{13}{3}\right\},\left\{1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$7560\,c^2\,d\,(b\,c-a\,d)^4\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{13}{3}\right\},\left\{1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$7182\,c\,d^2\,(b\,c-a\,d)^4\,x^{18}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{13}{3}\right\},\left\{1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$2268\,d^3\,(b\,c-a\,d)^4\,x^{21}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{13}{3}\right\},\left\{1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$378\,c^3\,(b\,c-a\,d)^4\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{13}{3}\right\},\left\{1,1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$1134\,c^2\,d\,(b\,c-a\,d)^4\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{13}{3}\right\},\left\{1,1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$1134\,c\,d^2\,(b\,c-a\,d)^4\,x^{18}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{13}{3}\right\},\left\{1,1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$378\,d^3\,(b\,c-a\,d)^4\,x^{21}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{13}{3}\right\},\left\{1,1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\Bigg)$$

Problem 98: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x^3)^{8/3}}{(c+d x^3)^2} \, dx$$

Optimal (type 3, 351 leaves, 5 steps):

$$\frac{b (2 b c-a d) x (a+b x^3)^{2/3}}{3 c d^2}-\frac{(b c-a d) x (a+b x^3)^{5/3}}{3 c d (c+d x^3)}-\frac{2 b^{5/3} (3 b c-4 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} d^3}+\frac{2 (b c-a d)^{5/3} (3 b c+a d) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^{5/3} d^3}+\frac{(b c-a d)^{5/3} (3 b c+a d) \text{Log}[c+d x^3]}{9 c^{5/3} d^3}-\frac{(b c-a d)^{5/3} (3 b c+a d) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-(a+b x^3)^{1/3}\right]}{3 c^{5/3} d^3}+\frac{b^{5/3} (3 b c-4 a d) \text{Log}\left[-b^{1/3} x+(a+b x^3)^{1/3}\right]}{3 d^3}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^2 x (a+b x^3)^{2/3} \text{AppellF1}\left[\frac{1}{3},-\frac{8}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^2 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 99: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x^3)^{5/3}}{(c+d x^3)^2} \, dx$$

Optimal (type 3, 301 leaves, 4 steps):

$$-\frac{(b c-a d) x (a+b x^3)^{2/3}}{3 c d (c+d x^3)}+\frac{b^{5/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2}-\frac{(b c-a d)^{2/3} (3 b c+2 a d) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^{5/3} d^2}-\frac{(b c-a d)^{2/3} (3 b c+2 a d) \text{Log}[c+d x^3]}{18 c^{5/3} d^2}+\frac{(b c-a d)^{2/3} (3 b c+2 a d) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-(a+b x^3)^{1/3}\right]}{6 c^{5/3} d^2}-\frac{b^{5/3} \text{Log}\left[-b^{1/3} x+(a+b x^3)^{1/3}\right]}{2 d^2}$$

Result (type 6, 60 leaves, 2 steps):

$$\frac{a x (a+b x^3)^{2/3} \text{AppellF1}\left[\frac{1}{3},-\frac{5}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^2 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 100: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{\left(c+d x^3\right)^2} \, dx$$

Optimal (type 3, 182 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^{2/3}}{3 c \left(c+d x^3\right)}+\frac{2 a \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^{5/3} (b c-a d)^{1/3}}+\frac{a \operatorname{Log}\left[c+d x^3\right]}{9 c^{5/3} (b c-a d)^{1/3}}-\frac{a \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{3 c^{5/3} (b c-a d)^{1/3}}$$

Result (type 3, 241 leaves, 8 steps):

$$\frac{x \left(a+b x^3\right)^{2/3}}{3 c \left(c+d x^3\right)}+\frac{2 a \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{3 \sqrt{3} c^{5/3} (b c-a d)^{1/3}}-\frac{2 a \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 c^{5/3} (b c-a d)^{1/3}}+\frac{a \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 c^{5/3} (b c-a d)^{1/3}}$$

Problem 101: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)^2} \, dx$$

Optimal (type 3, 217 leaves, 2 steps):

$$-\frac{d x \left(a+b x^3\right)^{2/3}}{3 c (b c-a d) \left(c+d x^3\right)}+\frac{(3 b c-2 a d) \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^{5/3} (b c-a d)^{4/3}}+\frac{(3 b c-2 a d) \operatorname{Log}\left[c+d x^3\right]}{18 c^{5/3} (b c-a d)^{4/3}}-\frac{(3 b c-2 a d) \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{6 c^{5/3} (b c-a d)^{4/3}}$$

Result (type 3, 276 leaves, 8 steps):

$$-\frac{d x \left(a+b x^3\right)^{2/3}}{3 c (b c-a d) \left(c+d x^3\right)}+\frac{(3 b c-2 a d) \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{3 \sqrt{3} c^{5/3} (b c-a d)^{4/3}}-$$

$$\frac{(3 b c-2 a d) \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 c^{5/3} (b c-a d)^{4/3}}+\frac{(3 b c-2 a d) \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{18 c^{5/3} (b c-a d)^{4/3}}$$

Problem 102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)^2} \, dx$$

Optimal (type 3, 261 leaves, 4 steps):

$$\frac{b (3 b c+a d) x}{3 a c (b c-a d)^2 \left(a+b x^3\right)^{1/3}}-\frac{d x}{3 c (b c-a d) \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)}-\frac{2 d (3 b c-a d) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^{5/3} (b c-a d)^{7/3}}-\frac{d (3 b c-a d) \text{Log}\left[c+d x^3\right]}{9 c^{5/3} (b c-a d)^{7/3}}+\frac{d (3 b c-a d) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{3 c^{5/3} (b c-a d)^{7/3}}$$

Result (type 5, 625 leaves, 2 steps):

$$-\frac{1}{420 (b c-a d)^2 x^5 \left(c+d x^3\right)} c \left(a+b x^3\right)^{2/3} \left(6860+\frac{13720 d x^3}{c}+\frac{6300 d^2 x^6}{c^2}-\frac{525 (b c-a d) x^3}{c \left(a+b x^3\right)}-\frac{1890 d (b c-a d) x^6}{c^2 \left(a+b x^3\right)}-\frac{945 d^2 (b c-a d) x^9}{c^3 \left(a+b x^3\right)}-6860 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-\frac{13720 d x^3 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]}{c}-\frac{6300 d^2 x^6 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]}{c^2}+\frac{2240 (b c-a d) x^3 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]}{c \left(a+b x^3\right)}+\frac{5320 d (b c-a d) x^6 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]}{c^2 \left(a+b x^3\right)}+\frac{2520 d^2 (b c-a d) x^9 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]}{c^3 \left(a+b x^3\right)}-\frac{54 (b c-a d)^3 x^9 \text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]}{c^3 \left(a+b x^3\right)^3}-\frac{108 d (b c-a d)^3 x^{12} \text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]}{c^4 \left(a+b x^3\right)^3}-\frac{54 d^2 (b c-a d)^3 x^{15} \text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]}{c^5 \left(a+b x^3\right)^3}\right)$$

Problem 103: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{7/3} \left(c+d x^3\right)^2} \, dx$$

Optimal (type 3, 324 leaves, 5 steps):

$$\frac{b\,(3 b c+4 a d)\,x}{12 a c\,(b c-a d)^2\,\left(a+b x^3\right)^{4/3}}+\frac{b\,\left(9 b^2 c^2-33 a b c d-4 a^2 d^2\right)x}{12 a^2 c\,(b c-a d)^3\,\left(a+b x^3\right)^{1/3}}-\frac{d x}{3 c\,(b c-a d)\,\left(a+b x^3\right)^{4/3}\left(c+d x^3\right)}+$$

$$\frac{d^2\,(9 b c-2 a d)\,\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, c^{5/3}\,(b c-a d)^{10/3}}+\frac{d^2\,(9 b c-2 a d)\,\text{Log}\left[c+d x^3\right]}{18 c^{5/3}\,(b c-a d)^{10/3}}-\frac{d^2\,(9 b c-2 a d)\,\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{6 c^{5/3}\,(b c-a d)^{10/3}}$$

Result (type 5, 1214 leaves, 2 steps):

$$\frac{1}{21840\, c^5 (b c - a d)^3 x^8 (a + b x^3)^{10/3} (c + d x^3)}$$

$$\Big(26130\, c^5 (b c - a d)^2 x^6 (a + b x^3)^2 + 89505\, c^4 d (b c - a d)^2 x^9 (a + b x^3)^2 + 84240\, c^3 d^2 (b c - a d)^2 x^{12} (a + b x^3)^2 +$$

$$26325\, c^2 d^3 (b c - a d)^2 x^{15} (a + b x^3)^2 + 748020\, c^6 (b c - a d) x^3 (a + b x^3)^3 + 2113020\, c^5 d (b c - a d) x^6 (a + b x^3)^3 +$$

$$1916460\, c^4 d^2 (b c - a d) x^9 (a + b x^3)^3 + 589680\, c^3 d^3 (b c - a d) x^{12} (a + b x^3)^3 - 2002000\, c^7 (a + b x^3)^4 - 5460000\, c^6 d x^3 (a + b x^3)^4 -$$

$$4914000\, c^5 d^2 x^6 (a + b x^3)^4 - 1506960\, c^4 d^3 x^9 (a + b x^3)^4 - 1248520\, c^6 (b c - a d) x^3 (a + b x^3)^3 \text{Hypergeometric2F1}\Big[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] -$$

$$3478020\, c^5 d (b c - a d) x^6 (a + b x^3)^3 \text{Hypergeometric2F1}\Big[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] - 3144960\, c^4 d^2 (b c - a d) x^9 (a + b x^3)^3$$

$$\text{Hypergeometric2F1}\Big[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] - 966420\, c^3 d^3 (b c - a d) x^{12} (a + b x^3)^3 \text{Hypergeometric2F1}\Big[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] +$$

$$2002000\, c^7 (a + b x^3)^4 \text{Hypergeometric2F1}\Big[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] + 5460000\, c^6 d x^3 (a + b x^3)^4 \text{Hypergeometric2F1}\Big[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] +$$

$$4914000\, c^5 d^2 x^6 (a + b x^3)^4 \text{Hypergeometric2F1}\Big[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] + 1506960\, c^4 d^3 x^9 (a + b x^3)^4$$

$$\text{Hypergeometric2F1}\Big[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] + 7938\, c^3 (b c - a d)^4 x^{12} \text{HypergeometricPFQ}\Big[\Big\{2, 2, \frac{10}{3}\Big\}, \Big\{1, \frac{16}{3}\Big\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] +$$

$$22680\, c^2 d (b c - a d)^4 x^{15} \text{HypergeometricPFQ}\Big[\Big\{2, 2, \frac{10}{3}\Big\}, \Big\{1, \frac{16}{3}\Big\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] +$$

$$21546\, c d^2 (b c - a d)^4 x^{18} \text{HypergeometricPFQ}\Big[\Big\{2, 2, \frac{10}{3}\Big\}, \Big\{1, \frac{16}{3}\Big\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] +$$

$$6804\, d^3 (b c - a d)^4 x^{21} \text{HypergeometricPFQ}\Big[\Big\{2, 2, \frac{10}{3}\Big\}, \Big\{1, \frac{16}{3}\Big\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] +$$

$$1134\, c^3 (b c - a d)^4 x^{12} \text{HypergeometricPFQ}\Big[\Big\{2, 2, 2, \frac{10}{3}\Big\}, \Big\{1, 1, \frac{16}{3}\Big\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] +$$

$$3402\, c^2 d (b c - a d)^4 x^{15} \text{HypergeometricPFQ}\Big[\Big\{2, 2, 2, \frac{10}{3}\Big\}, \Big\{1, 1, \frac{16}{3}\Big\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] +$$

$$3402\, c d^2 (b c - a d)^4 x^{18} \text{HypergeometricPFQ}\Big[\Big\{2, 2, 2, \frac{10}{3}\Big\}, \Big\{1, 1, \frac{16}{3}\Big\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big] +$$

$$1134\, d^3 (b c - a d)^4 x^{21} \text{HypergeometricPFQ}\Big[\Big\{2, 2, 2, \frac{10}{3}\Big\}, \Big\{1, 1, \frac{16}{3}\Big\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\Big]\Big)$$

Problem 109: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{14/3}}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 541 leaves, 7 steps):

$$-\frac{b\,(2 b c-a d)\left(18 b^2 c^2-18 a b c d-5 a^2 d^2\right) x \left(a+b x^3\right)^{2/3}}{18 c^2 d^4}+\frac{b\left(18 b^2 c^2-10 a b c d-5 a^2 d^2\right) x \left(a+b x^3\right)^{5/3}}{18 c^2 d^3}-\frac{(b c-a d)\, x \left(a+b x^3\right)^{11/3}}{6 c d \left(c+d x^3\right)^2}-\frac{(b c-a d)(12 b c+5 a d)\, x \left(a+b x^3\right)^{8/3}}{18 c^2 d^2 \left(c+d x^3\right)}+\frac{b^{8/3}\left(54 b^2 c^2-126 a b c d+77 a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, d^5}-\frac{(b c-a d)^{8/3}\left(54 b^2 c^2+18 a b c d+5 a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, c^{8/3} d^5}-\frac{(b c-a d)^{8/3}\left(54 b^2 c^2+18 a b c d+5 a^2 d^2\right) \text{Log}\left[c+d x^3\right]}{54 c^{8/3} d^5}+\frac{(b c-a d)^{8/3}\left(54 b^2 c^2+18 a b c d+5 a^2 d^2\right) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} d^5}-\frac{b^{8/3}\left(54 b^2 c^2-126 a b c d+77 a^2 d^2\right) \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{18 d^5}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^4 x \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},-\frac{14}{3},3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^3 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 110: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{11/3}}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 458 leaves, 6 steps):

$$\frac{b\left(18 b^2 c^2-7 a b c d-5 a^2 d^2\right) x\left(a+b x^3\right)^{2/3}}{18 c^2 d^3}-\frac{(b c-a d) x\left(a+b x^3\right)^{8/3}}{6 c d\left(c+d x^3\right)^2}-$$

$$\frac{(b c-a d)(9 b c+5 a d) x\left(a+b x^3\right)^{5/3}}{18 c^2 d^2\left(c+d x^3\right)}-\frac{b^{8/3}(9 b c-11 a d) \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} d^4}+$$

$$\frac{(b c-a d)^{5/3}\left(27 b^2 c^2+12 a b c d+5 a^2 d^2\right) \operatorname{ArcTan}\left[\frac{1+\frac{2(b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} d^4}+\frac{(b c-a d)^{5/3}\left(27 b^2 c^2+12 a b c d+5 a^2 d^2\right) \operatorname{Log}\left[c+d x^3\right]}{54 c^{8/3} d^4}-$$

$$\frac{(b c-a d)^{5/3}\left(27 b^2 c^2+12 a b c d+5 a^2 d^2\right) \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} d^4}+\frac{b^{8/3}(9 b c-11 a d) \operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 d^4}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^3 x\left(a+b x^3\right)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3},-\frac{11}{3}, 3, \frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^3\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 111: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{8/3}}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 391 leaves, 5 steps):

$$-\frac{(b c-a d) x\left(a+b x^3\right)^{5/3}}{6 c d\left(c+d x^3\right)^2}-\frac{(b c-a d)(6 b c+5 a d) x\left(a+b x^3\right)^{2/3}}{18 c^2 d^2\left(c+d x^3\right)}+\frac{b^{8/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^3}-$$

$$\frac{(b c-a d)^{2/3}\left(9 b^2 c^2+6 a b c d+5 a^2 d^2\right) \operatorname{ArcTan}\left[\frac{1+\frac{2(b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} d^3}-\frac{(b c-a d)^{2/3}\left(9 b^2 c^2+6 a b c d+5 a^2 d^2\right) \operatorname{Log}\left[c+d x^3\right]}{54 c^{8/3} d^3}+$$

$$\frac{(b c-a d)^{2/3}\left(9 b^2 c^2+6 a b c d+5 a^2 d^2\right) \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} d^3}-\frac{b^{8/3} \operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 d^3}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^2 x \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},-\frac{8}{3},3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^3 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 112: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{5/3}}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 217 leaves, 3 steps):

$$\frac{x \left(a+b x^3\right)^{5/3}}{6 c \left(c+d x^3\right)^2}+\frac{5 a x \left(a+b x^3\right)^{2/3}}{18 c^2 \left(c+d x^3\right)}+\frac{5 a^2 \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c-a d)^{1/3}}+\frac{5 a^2 \text{Log}\left[c+d x^3\right]}{54 c^{8/3} (b c-a d)^{1/3}}-\frac{5 a^2 \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} (b c-a d)^{1/3}}$$

Result (type 3, 276 leaves, 9 steps):

$$\frac{x \left(a+b x^3\right)^{5/3}}{6 c \left(c+d x^3\right)^2}+\frac{5 a x \left(a+b x^3\right)^{2/3}}{18 c^2 \left(c+d x^3\right)}+\frac{5 a^2 \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{9 \sqrt{3} c^{8/3} (b c-a d)^{1/3}}-\frac{5 a^2 \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{27 c^{8/3} (b c-a d)^{1/3}}+\frac{5 a^2 \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{54 c^{8/3} (b c-a d)^{1/3}}$$

Problem 113: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 267 leaves, 3 steps):

$$-\frac{d x \left(a+b x^3\right)^{5/3}}{6 c (b c-a d) \left(c+d x^3\right)^2}+\frac{(6 b c-5 a d) x \left(a+b x^3\right)^{2/3}}{18 c^2 (b c-a d) \left(c+d x^3\right)}+\frac{a (6 b c-5 a d) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c-a d)^{4/3}}+$$
$$\frac{a (6 b c-5 a d) \text{Log}\left[c+d x^3\right]}{54 c^{8/3} (b c-a d)^{4/3}}-\frac{a (6 b c-5 a d) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} (b c-a d)^{4/3}}$$

Result (type 3, 326 leaves, 9 steps):

$$-\frac{d\,x\,\left(a+b\,x^3\right)^{5/3}}{6\,c\,\left(b\,c-a\,d\right)\,\left(c+d\,x^3\right)^2}+\frac{\left(6\,b\,c-5\,a\,d\right)\,x\,\left(a+b\,x^3\right)^{2/3}}{18\,c^2\,\left(b\,c-a\,d\right)\,\left(c+d\,x^3\right)}+\frac{a\,\left(6\,b\,c-5\,a\,d\right)\,\text{ArcTan}\left[\frac{c^{1/3}+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}\,c^{1/3}}\right]}{9\,\sqrt{3}\,c^{8/3}\,\left(b\,c-a\,d\right)^{4/3}}-\frac{a\,\left(6\,b\,c-5\,a\,d\right)\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{27\,c^{8/3}\,\left(b\,c-a\,d\right)^{4/3}}+\frac{a\,\left(6\,b\,c-5\,a\,d\right)\,\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{(a+b\,x^3)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{54\,c^{8/3}\,\left(b\,c-a\,d\right)^{4/3}}$$

Problem 114: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)^3}\,dx$$

Optimal (type 3, 307 leaves, 4 steps):

$$-\frac{d\,x\,\left(a+b\,x^3\right)^{2/3}}{6\,c\,\left(b\,c-a\,d\right)\,\left(c+d\,x^3\right)^2}-\frac{d\,\left(9\,b\,c-5\,a\,d\right)\,x\,\left(a+b\,x^3\right)^{2/3}}{18\,c^2\,\left(b\,c-a\,d\right)^2\,\left(c+d\,x^3\right)}+\frac{\left(9\,b^2\,c^2-12\,a\,b\,c\,d+5\,a^2\,d^2\right)\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,c^{8/3}\,\left(b\,c-a\,d\right)^{7/3}}+\frac{\left(9\,b^2\,c^2-12\,a\,b\,c\,d+5\,a^2\,d^2\right)\,\text{Log}\left[c+d\,x^3\right]}{54\,c^{8/3}\,\left(b\,c-a\,d\right)^{7/3}}-\frac{\left(9\,b^2\,c^2-12\,a\,b\,c\,d+5\,a^2\,d^2\right)\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{18\,c^{8/3}\,\left(b\,c-a\,d\right)^{7/3}}$$

Result (type 5, 167 leaves, 2 steps):

$$-\left(\left(x\,\left(c\,d\,\left(3\,b^2\,c\,x^3\,\left(4\,c+3\,d\,x^3\right)-a^2\,d\,\left(8\,c+5\,d\,x^3\right)+a\,b\,\left(12\,c^2+c\,d\,x^3-5\,d^2\,x^6\right)\right)-2\,\left(9\,b^2\,c^2-12\,a\,b\,c\,d+5\,a^2\,d^2\right)\,\left(c+d\,x^3\right)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]\right)\right)\Big/\left(18\,c^3\,\left(b\,c-a\,d\right)^2\,\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)^2\right)\right)$$

Problem 115: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x^3\right)^{4/3}\,\left(c+d\,x^3\right)^3}\,dx$$

Optimal (type 3, 377 leaves, 5 steps):

$$-\frac{d\,x}{6\,c\,(b\,c-a\,d)\,(a+b\,x^3)^{1/3}\,(c+d\,x^3)^2}+\frac{b\,(6\,b\,c+a\,d)\,x}{6\,a\,c\,(b\,c-a\,d)^2\,(a+b\,x^3)^{1/3}\,(c+d\,x^3)}+$$

$$\frac{d\,(18\,b^2\,c^2+15\,a\,b\,c\,d-5\,a^2\,d^2)\,x\,(a+b\,x^3)^{2/3}}{18\,a\,c^2\,(b\,c-a\,d)^3\,(c+d\,x^3)}-\frac{d\,(27\,b^2\,c^2-18\,a\,b\,c\,d+5\,a^2\,d^2)\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,c^{8/3}\,(b\,c-a\,d)^{10/3}}-$$

$$\frac{d\,(27\,b^2\,c^2-18\,a\,b\,c\,d+5\,a^2\,d^2)\,\text{Log}[c+d\,x^3]}{54\,c^{8/3}\,(b\,c-a\,d)^{10/3}}+\frac{d\,(27\,b^2\,c^2-18\,a\,b\,c\,d+5\,a^2\,d^2)\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\right]}{18\,c^{8/3}\,(b\,c-a\,d)^{10/3}}$$

Result (type 5, 428 leaves, 2 steps):

$$-\frac{1}{16\,380\,c^5\,(b\,c-a\,d)^3\,x^8\,(a+b\,x^3)^{7/3}\,(c+d\,x^3)^2}$$
$$\Big(65\,c^2\,(a+b\,x^3)^2\,\big(14\,000\,a^2\,c^5+21\,896\,a\,b\,c^5\,x^3+48\,104\,a^2\,c^4\,d\,x^3+8391\,b^2\,c^5\,x^6+70\,802\,a\,b\,c^4\,d\,x^6+60\,807\,a^2\,c^3\,d^2\,x^6+24\,417\,b^2\,c^4\,d\,x^9+$$
$$81\,534\,a\,b\,c^3\,d^2\,x^9+33\,657\,a^2\,c^2\,d^3\,x^9+23\,409\,b^2\,c^3\,d^2\,x^{12}+38\,652\,a\,b\,c^2\,d^3\,x^{12}+7155\,a^2\,c\,d^4\,x^{12}+7425\,b^2\,c^2\,d^3\,x^{15}+5940\,a\,b\,c\,d^4\,x^{15}+$$
$$243\,a^2\,d^5\,x^{15}-28\,(c+d\,x^3)^2\,(27\,b^2\,c^2\,x^6\,(7\,c+6\,d\,x^3)+9\,a\,b\,c\,x^3\,(73\,c^2+104\,c\,d\,x^3+33\,d^2\,x^6)+a^2\,(500\,c^3+843\,c^2\,d\,x^3+375\,c\,d^2\,x^6+27\,d^3\,x^9))$$
$$\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\big)-$$
$$486\,(b\,c-a\,d)^4\,x^{12}\,(c+d\,x^3)^3\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\Big)$$

Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{(a+b\,x^3)^{7/3}\,(c+d\,x^3)^3}\,dx$$

Optimal (type 3, 463 leaves, 6 steps):

$$-\frac{d\,x}{6\,c\,(b\,c-a\,d)\,(a+b\,x^3)^{4/3}\,(c+d\,x^3)^2}+\frac{b\,(3\,b\,c+2\,a\,d)\,x}{12\,a\,c\,(b\,c-a\,d)^2\,(a+b\,x^3)^{4/3}\,(c+d\,x^3)}+\frac{b\,(9\,b^2\,c^2-42\,a\,b\,c\,d-2\,a^2\,d^2)\,x}{12\,a^2\,c\,(b\,c-a\,d)^3\,(a+b\,x^3)^{1/3}\,(c+d\,x^3)}+$$

$$\frac{d\,(27\,b^3\,c^3-135\,a\,b^2\,c^2\,d-42\,a^2\,b\,c\,d^2+10\,a^3\,d^3)\,x\,(a+b\,x^3)^{2/3}}{36\,a^2\,c^2\,(b\,c-a\,d)^4\,(c+d\,x^3)}+\frac{d^2\,(54\,b^2\,c^2-24\,a\,b\,c\,d+5\,a^2\,d^2)\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,c^{8/3}\,(b\,c-a\,d)^{13/3}}+$$

$$\frac{d^2\,(54\,b^2\,c^2-24\,a\,b\,c\,d+5\,a^2\,d^2)\,\text{Log}[c+d\,x^3]}{54\,c^{8/3}\,(b\,c-a\,d)^{13/3}}-\frac{d^2\,(54\,b^2\,c^2-24\,a\,b\,c\,d+5\,a^2\,d^2)\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\right]}{18\,c^{8/3}\,(b\,c-a\,d)^{13/3}}$$

Result (type 5, 1990 leaves, 2 steps) :

$$-\frac{1}{524160\,c^6\,(b\,c-a\,d)^4\,x^{11}\,(a+b\,x^3)^{10/3}\,(c+d\,x^3)^2}$$

$$\Big(522756\,c^6\,(b\,c-a\,d)^3\,x^9\,(a+b\,x^3)^2+1516320\,c^5\,d\,(b\,c-a\,d)^3\,x^{12}\,(a+b\,x^3)^2+2198664\,c^4\,d^2\,(b\,c-a\,d)^3\,x^{15}\,(a+b\,x^3)^2+$$

$$1415232\,c^3\,d^3\,(b\,c-a\,d)^3\,x^{18}\,(a+b\,x^3)^2+341172\,c^2\,d^4\,(b\,c-a\,d)^3\,x^{21}\,(a+b\,x^3)^2+28042560\,c^7\,(b\,c-a\,d)^2\,x^6\,(a+b\,x^3)^3+$$
$$107602560\,c^6\,d\,(b\,c-a\,d)^2\,x^9\,(a+b\,x^3)^3+157697280\,c^5\,d^2\,(b\,c-a\,d)^2\,x^{12}\,(a+b\,x^3)^3+101088000\,c^4\,d^3\,(b\,c-a\,d)^2\,x^{15}\,(a+b\,x^3)^3+$$
$$24261120\,c^3\,d^4\,(b\,c-a\,d)^2\,x^{18}\,(a+b\,x^3)^3-265470660\,c^8\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)^4-1019636800\,c^7\,d\,(b\,c-a\,d)\,x^6\,(a+b\,x^3)^4-$$
$$1466086440\,c^6\,d^2\,(b\,c-a\,d)\,x^9\,(a+b\,x^3)^4-930252960\,c^5\,d^3\,(b\,c-a\,d)\,x^{12}\,(a+b\,x^3)^4-221899860\,c^4\,d^4\,(b\,c-a\,d)\,x^{15}\,(a+b\,x^3)^4+$$
$$335877360\,c^9\,(a+b\,x^3)^5+1279532800\,c^8\,d\,x^3\,(a+b\,x^3)^5+1823334240\,c^7\,d^2\,x^6\,(a+b\,x^3)^5+1151579520\,c^6\,d^3\,x^9\,(a+b\,x^3)^5+$$
$$273939120\,c^5\,d^4\,x^{12}\,(a+b\,x^3)^5-67420080\,c^7\,(b\,c-a\,d)^2\,x^6\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$259692160\,c^6\,d\,(b\,c-a\,d)^2\,x^9\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$377700960\,c^5\,d^2\,(b\,c-a\,d)^2\,x^{12}\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$241113600\,c^4\,d^3\,(b\,c-a\,d)^2\,x^{15}\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$57723120\,c^3\,d^4\,(b\,c-a\,d)^2\,x^{18}\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$349440000\,c^8\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$1339520000\,c^7\,d\,(b\,c-a\,d)\,x^6\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$

$$1\,921\,920\,000\, c^6 d^2 (b c - a d) x^9 (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$1\,218\,147\,840\, c^5 d^3 (b c - a d) x^{12} (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$290\,384\,640\, c^4 d^4 (b c - a d) x^{15} (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$335\,877\,360\, c^9 (a + b x^3)^5 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 1\,279\,532\,800\, c^8 d x^3 (a + b x^3)^5$$

$$\text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 1\,823\,334\,240\, c^7 d^2 x^6 (a + b x^3)^5 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$1\,151\,579\,520\, c^6 d^3 x^9 (a + b x^3)^5 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 273\,939\,120\, c^5 d^4 x^{12} (a + b x^3)^5$$

$$\text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 57\,834\, c^4 (b c - a d)^5 x^{15} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$224\,532\, c^3 d (b c - a d)^5 x^{18} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$326\,592\, c^2 d^2 (b c - a d)^5 x^{21} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$210\,924\, c d^3 (b c - a d)^5 x^{24} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$51\,030\, d^4 (b c - a d)^5 x^{27} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$5103\, c^4 (b c - a d)^5 x^{15} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$20\,412\, c^3 d (b c - a d)^5 x^{18} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$30\,618\, c^2 d^2 (b c - a d)^5 x^{21} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$20\,412\, c d^3 (b c - a d)^5 x^{24} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$5103\, d^4 (b c - a d)^5 x^{27} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]\Bigg)$$

Test results for the 1081 problems in "1.1.3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q.m"

Problem 455: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 256 leaves, ? steps):

$$\frac{2 x \left(4 c+d x^3\right)}{81 c d^2 \left(8 c-d x^3\right) \sqrt{c+d x^3}}-\frac{2 \sqrt{2+\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]}{81\times 3^{1/4} c d^{7/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7 \sqrt{1+\frac{d x^3}{c}} \text{AppellF1}\left[\frac{7}{3},2,\frac{3}{2},\frac{10}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{448 c^3 \sqrt{c+d x^3}}$$

Problem 574: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 268 leaves, 6 steps):

$$-\frac{7 a x^2 \left(a+b x^3\right)^{1/3}}{18 b^2 d}-\frac{x^5 \left(a+b x^3\right)^{1/3}}{6 b d}+\frac{11 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3} d}-\frac{2^{1/3} a^2 \text{ArcTan}\left[\frac{1+\frac{2\cdot 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{8/3} d}+$$
$$\frac{a^2 \text{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} b^{8/3} d}+\frac{11 a^2 \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{18 b^{8/3} d}-\frac{a^2 \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} b^{8/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^8 \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{8}{3},-\frac{1}{3},1,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{8 a d \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 575: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 233 leaves, 5 steps):

$$-\frac{x^2 \left(a+b x^3\right)^{1/3}}{3 b d}+\frac{4 a \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3} d}-\frac{2^{1/3} a \operatorname{ArcTan}\left[\frac{1+\frac{2\ 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{5/3} d}+\frac{a \operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} b^{5/3} d}+\frac{2 a \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{3 b^{5/3} d}-\frac{a \operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} b^{5/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^5 \left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[\frac{5}{3},-\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{5 a d \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 576: Result unnecessarily involves higher level functions.

$$\int \frac{x \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 201 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3} d}-\frac{2^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2\ 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3} d}+\frac{\operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} b^{2/3} d}+\frac{\operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 b^{2/3} d}-\frac{\operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} b^{2/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^2 \left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{2 a d \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 577: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^2 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 156 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{a d x}-\frac{2^{1/3} b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a d}+\frac{b^{1/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} a d}-\frac{b^{1/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} a d}$$

Result (type 5, 77 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{1/3} \left(1-\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[-\frac{1}{3},-\frac{1}{3},\frac{2}{3},-\frac{2 b x^3}{a-b x^3}\right]}{a d x \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 578: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^5 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 183 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{4 a d x^4}-\frac{5 b \left(a+b x^3\right)^{1/3}}{4 a^2 d x}-\frac{2^{1/3} b^{4/3} \text{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^2 d}+\frac{b^{4/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} a^2 d}-\frac{b^{4/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} a^2 d}$$

Result (type 5, 117 leaves, 2 steps):

$$-\frac{1}{4 a^2 d x^4 \left(a+b x^3\right)^{2/3}} \left(a^2+4 a b x^3+3 b^2 x^6-b x^3 \left(a+3 b x^3\right) \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2 b x^3}{a+b x^3}\right]+3 b x^3 \left(a-b x^3\right) \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2 b x^3}{a+b x^3}\right]\right)$$

Problem 579: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^8 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{7 a d x^7}-\frac{2 b \left(a+b x^3\right)^{1/3}}{7 a^2 d x^4}-\frac{8 b^2 \left(a+b x^3\right)^{1/3}}{7 a^3 d x}-\frac{2^{1/3} b^{7/3} \text{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^3 d}+\frac{b^{7/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} a^3 d}-\frac{b^{7/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} a^3 d}$$

Result (type 5, 244 leaves, 2 steps):

$$-\frac{1}{28\,a^3\,d\,x^7\,\left(a+b\,x^3\right)^{2/3}}\left(4\,a^3+10\,a^2\,b\,x^3+24\,a\,b^2\,x^6+18\,b^3\,x^9-2\,b\,x^3\,\left(2\,a^2+3\,a\,b\,x^3+9\,b^2\,x^6\right)\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+\right.$$
$$15\,a^2\,b\,x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+12\,a\,b^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-$$
$$\left.27\,b^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-9\,b\,x^3\,\left(a-b\,x^3\right)^2\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{5}{3}\right\},\frac{2\,b\,x^3}{a+b\,x^3}\right]\right)$$

Problem 580: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x^{11}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Optimal (type 3, 237 leaves, 6 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{10\,a\,d\,x^{10}}-\frac{11\,b\,\left(a+b\,x^3\right)^{1/3}}{70\,a^2\,d\,x^7}-\frac{37\,b^2\,\left(a+b\,x^3\right)^{1/3}}{140\,a^3\,d\,x^4}-\frac{169\,b^3\,\left(a+b\,x^3\right)^{1/3}}{140\,a^4\,d\,x}-$$
$$\frac{2^{1/3}\,b^{10/3}\,\text{ArcTan}\left[\frac{1+\frac{2\cdot2^{1/3}\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,a^4\,d}+\frac{b^{10/3}\,\text{Log}\left[a\,d-b\,d\,x^3\right]}{3\times2^{2/3}\,a^4\,d}-\frac{b^{10/3}\,\text{Log}\left[2^{1/3}\,b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,a^4\,d}$$

Result (type 5, 423 leaves, 2 steps):

$$-\frac{1}{280\,a^4\,d\,x^{10}\,\left(a+b\,x^3\right)^{2/3}}\left(28\,a^4+64\,a^3\,b\,x^3+90\,a^2\,b^2\,x^6+216\,a\,b^3\,x^9+162\,b^4\,x^{12}-28\,a^3\,b\,x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-\right.$$
$$36\,a^2\,b^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-54\,a\,b^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-$$
$$162\,b^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+117\,a^3\,b\,x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+$$
$$99\,a^2\,b^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+81\,a\,b^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-$$
$$297\,b^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-54\,b\,x^3\,\left(a-b\,x^3\right)^2\,\left(2\,a+3\,b\,x^3\right)\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{5}{3}\right\},\frac{2\,b\,x^3}{a+b\,x^3}\right]+$$
$$\left.27\,b\,x^3\,\left(a-b\,x^3\right)^3\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{5}{3}\right\},\frac{2\,b\,x^3}{a+b\,x^3}\right]\right)$$

Problem 581: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 5, 521 leaves, 22 steps):

$$-\frac{3 a x \left(a+b x^3\right)^{1/3}}{5 b^2 d}-\frac{x^4 \left(a+b x^3\right)^{1/3}}{5 b d}-\frac{2^{1/3} a^{5/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{7/3} d}-\frac{a^{5/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, b^{7/3} d}-\frac{2 a^2 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{5 b^2 d \left(a+b x^3\right)^{2/3}}-\frac{a^{5/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} b^{7/3} d}+\frac{a^{5/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} b^{7/3} d}-\frac{2^{1/3} a^{5/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{7/3} d}+\frac{a^{5/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} b^{7/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7 \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{7}{3},-\frac{1}{3},1,\frac{10}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{7 a d \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 582: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 5, 494 leaves, 21 steps):

$$-\frac{x\left(a+b x^3\right)^{1/3}}{2 b d}-\frac{2^{1/3} a^{2/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{4/3} d}-\frac{a^{2/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, b^{4/3} d}-$$

$$\frac{a x\left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 b d\left(a+b x^3\right)^{2/3}}-\frac{a^{2/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} b^{4/3} d}+$$

$$\frac{a^{2/3} \text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} b^{4/3} d}-\frac{2^{1/3} a^{2/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{4/3} d}+\frac{a^{2/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} b^{4/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^4\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{4 a d\left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 583: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{a d-b d x^3}\, dx$$

Optimal (type 3, 416 leaves, 14 steps):

$$-\frac{2^{1/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{1/3} b^{1/3} d}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, a^{1/3} b^{1/3} d}-\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{1/3} b^{1/3} d}+$$

$$\frac{\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{1/3} b^{1/3} d}-\frac{2^{1/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{1/3} b^{1/3} d}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{1/3} b^{1/3} d}$$

Result (type 6, 61 leaves, 2 steps):

$$\frac{x\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{a d\left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 584: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^3 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 5, 496 leaves, 21 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{2 a d x^2}-\frac{2^{1/3} b^{2/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{4/3} d}-\frac{b^{2/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, a^{4/3} d}+\frac{b x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 a d \left(a+b x^3\right)^{2/3}}-\frac{b^{2/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{4/3} d}+\frac{b^{2/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{4/3} d}-\frac{2^{1/3} b^{2/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{4/3} d}+\frac{b^{2/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{4/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[-\frac{2}{3},-\frac{1}{3},1,\frac{1}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{2 a d x^2 \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 585: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^6 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 5, 523 leaves, 22 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{5 a d x^5}-\frac{3 b \left(a+b x^3\right)^{1/3}}{5 a^2 d x^2}-\frac{2^{1/3} b^{5/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{7/3} d}-$$

$$\frac{b^{5/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, a^{7/3} d}+\frac{2 b^2 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{5 a^2 d \left(a+b x^3\right)^{2/3}}-\frac{b^{5/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{7/3} d}+$$

$$\frac{b^{5/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{7/3} d}-\frac{2^{1/3} b^{5/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{7/3} d}+\frac{b^{5/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{7/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[-\frac{5}{3},-\frac{1}{3},1,-\frac{2}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{5 a d x^5 \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 593: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 264 leaves, 5 steps):

$$-\frac{4 a x \left(a+b x^3\right)^{2/3}}{9 b^2 d}-\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 b d}-\frac{14 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{7/3} d}+\frac{2^{2/3} a^2 \text{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{7/3} d}+$$

$$\frac{a^2 \text{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} b^{7/3} d}-\frac{a^2 \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^{7/3} d}+\frac{7 a^2 \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{9 b^{7/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{7}{3},-\frac{2}{3},1,\frac{10}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{7 a d \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 594: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 229 leaves, 4 steps):

$$-\frac{x\left(a+b x^3\right)^{2/3}}{3 b d}-\frac{5 a \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{4/3} d}+\frac{2^{2/3} a \operatorname{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{4/3} d}+\frac{a \operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} b^{4/3} d}-\frac{a \operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^{4/3} d}+\frac{5 a \operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 b^{4/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^4\left(a+b x^3\right)^{2/3} \operatorname{AppellF1}\left[\frac{4}{3},-\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{4 a d\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 595: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{a d-b d x^3}\, dx$$

Optimal (type 3, 200 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3} d}+\frac{2^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3} d}+\frac{\operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} b^{1/3} d}-\frac{\operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^{1/3} d}+\frac{\operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 b^{1/3} d}$$

Result (type 6, 61 leaves, 2 steps):

$$\frac{x\left(a+b x^3\right)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{a d\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 596: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^3\left(a d-b d x^3\right)}\, dx$$

Optimal (type 3, 157 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{2 a d x^2}+\frac{2^{2/3} b^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a d}+\frac{b^{2/3} \operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} a d}-\frac{b^{2/3} \operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} a d}$$

Result (type 5, 79 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{2/3} \left(1-\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\frac{2 b x^3}{a-b x^3}\right]}{2 a d x^2 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 597: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^6 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 182 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{5 a d x^5}-\frac{7 b \left(a+b x^3\right)^{2/3}}{10 a^2 d x^2}+\frac{2^{2/3} b^{5/3} \text{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^2 d}+\frac{b^{5/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} a^2 d}-\frac{b^{5/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} a^2 d}$$

Result (type 5, 121 leaves, 2 steps):

$$-\frac{1}{10 a^2 d x^5 \left(a+b x^3\right)^{1/3}} \left(2 a^2+5 a b x^3+3 b^2 x^6-4 b x^3 \left(2 a+3 b x^3\right) \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]+12 b x^3 \left(a-b x^3\right) \text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]\right)$$

Problem 598: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^9 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 209 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{8 a d x^8}-\frac{b \left(a+b x^3\right)^{2/3}}{4 a^2 d x^5}-\frac{5 b^2 \left(a+b x^3\right)^{2/3}}{8 a^3 d x^2}+\frac{2^{2/3} b^{8/3} \text{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^3 d}+\frac{b^{8/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} a^3 d}-\frac{b^{8/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} a^3 d}$$

Result (type 5, 244 leaves, 2 steps):

$$-\frac{1}{40 a^3 d x^8 \left(a+b x^3\right)^{1/3}} \left(5 a^3+11 a^2 b x^3+15 a b^2 x^6+9 b^3 x^9-4 b x^3 \left(5 a^2+6 a b x^3+9 b^2 x^6\right) \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]+\right.$$
$$42 a^2 b x^3 \text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]+12 a b^2 x^6 \text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]-$$
$$\left.54 b^3 x^9 \text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]-18 b x^3 \left(a-b x^3\right)^2 \text{HypergeometricPFQ}\left[\left\{\frac{1}{3},2,2\right\},\left\{1,\frac{4}{3}\right\},\frac{2 b x^3}{a+b x^3}\right]\right)$$

Problem 599: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^{12} \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 236 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{11 a d x^{11}}-\frac{13 b \left(a+b x^3\right)^{2/3}}{88 a^2 d x^8}-\frac{49 b^2 \left(a+b x^3\right)^{2/3}}{220 a^3 d x^5}-\frac{293 b^3 \left(a+b x^3\right)^{2/3}}{440 a^4 d x^2}+\frac{2^{2/3} b^{11/3} \text{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^4 d}+\frac{b^{11/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} a^4 d}-\frac{b^{11/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} a^4 d}$$

Result (type 5, 391 leaves, 2 steps):

$$-\frac{1}{440 a^4 d x^{11} \left(a+b x^3\right)^{1/3}}\left(40 a^4+85 a^3 b x^3+99 a^2 b^2 x^6+135 a b^3 x^9+81 b^4 x^{12}-\right.$$
$$160 a^3 b x^3 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]-180 a^2 b^2 x^6 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]-$$
$$216 a b^3 x^9 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]-324 b^4 x^{12} \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]+$$
$$396 a^3 b x^3 \text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]+198 a^2 b^2 x^6 \text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]-$$
$$594 b^4 x^{12} \text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]-54 b x^3 \left(a-b x^3\right)^2 \left(5 a+6 b x^3\right) \text{HypergeometricPFQ}\left[\left\{\frac{1}{3},2,2\right\},\left\{1,\frac{4}{3}\right\},\frac{2 b x^3}{a+b x^3}\right]+$$
$$\left.54 b x^3 \left(a-b x^3\right)^3 \text{HypergeometricPFQ}\left[\left\{\frac{1}{3},2,2,2\right\},\left\{1,1,\frac{4}{3}\right\},\frac{2 b x^3}{a+b x^3}\right]\right)$$

Problem 600: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 5, 512 leaves, 14 steps):

$$-\frac{9 a x^2 \left(a+b x^3\right)^{2/3}}{28 b^2 d}-\frac{x^5 \left(a+b x^3\right)^{2/3}}{7 b d}+\frac{2^{2/3} a^{7/3} \text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{8/3} d}+\frac{a^{7/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}\, b^{8/3} d}-$$

$$\frac{19 a^2 x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{28 b^2 d \left(a+b x^3\right)^{1/3}}+\frac{a^{7/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2 \left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} b^{8/3} d}+$$

$$\frac{a^{7/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} b^{8/3} d}-\frac{2^{2/3} a^{7/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{8/3} d}-\frac{a^{7/3} \text{Log}\left[\frac{b^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3} \left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} b^{8/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^8 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{8}{3},-\frac{2}{3},1,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{8 a d \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 601: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 5, 485 leaves, 13 steps):

$$-\frac{x^2 \left(a+b x^3\right)^{2/3}}{4 b d}+\frac{2^{2/3} a^{4/3} \text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{5/3} d}+\frac{a^{4/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}\, b^{5/3} d}-$$

$$\frac{3 a x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{4 b d \left(a+b x^3\right)^{1/3}}+\frac{a^{4/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2 \left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} b^{5/3} d}+$$

$$\frac{a^{4/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} b^{5/3} d}-\frac{2^{2/3} a^{4/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{5/3} d}-\frac{a^{4/3} \text{Log}\left[\frac{b^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3} \left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} b^{5/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^5 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{5 a d \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 602: Result unnecessarily involves higher level functions.

$$\int \frac{x\left(a+b x^3\right)^{2/3}}{a d-b d x^3}\,dx$$

Optimal (type 5, 457 leaves, 11 steps):

$$\frac{2^{2/3} a^{1/3} \text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3} d}+\frac{a^{1/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}\, b^{2/3} d}-\frac{x^2\left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{2 d\left(a+b x^3\right)^{1/3}}+\frac{a^{1/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2\left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} b^{2/3} d}+$$
$$\frac{a^{1/3} \text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} b^{2/3} d}-\frac{2^{2/3} a^{1/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{2/3} d}-\frac{a^{1/3} \text{Log}\left[\frac{b^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3}\left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} b^{2/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^2\left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{2 a d\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 603: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^2\left(a d-b d x^3\right)}\,dx$$

Optimal (type 5, 483 leaves, 13 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{a d x}+\frac{2^{2/3} b^{1/3} \text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{2/3} d}+\frac{b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}\, a^{2/3} d}+$$
$$\frac{b x^2\left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{2 a d\left(a+b x^3\right)^{1/3}}+\frac{b^{1/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2\left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} a^{2/3} d}+$$
$$\frac{b^{1/3} \text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} a^{2/3} d}-\frac{2^{2/3} b^{1/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{2/3} d}-\frac{b^{1/3} \text{Log}\left[\frac{b^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3}\left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} a^{2/3} d}$$

Result (type 6, 64 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{2/3} \text{AppellF1}\left[-\frac{1}{3},-\frac{2}{3},1,\frac{2}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{a d x \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 604: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^5 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 5, 512 leaves, 14 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{4 a d x^4}-\frac{3 b \left(a+b x^3\right)^{2/3}}{2 a^2 d x}+\frac{2^{2/3} b^{4/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{5/3} d}+\frac{b^{4/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}\, a^{5/3} d}+$$

$$\frac{3 b^2 x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{4 a^2 d \left(a+b x^3\right)^{1/3}}+\frac{b^{4/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2 \left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6 \times 2^{1/3} a^{5/3} d}+$$

$$\frac{b^{4/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 \times 2^{1/3} a^{5/3} d}-\frac{2^{2/3} b^{4/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{5/3} d}-\frac{b^{4/3} \text{Log}\left[\frac{b^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3} \left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2 \times 2^{1/3} a^{5/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{2/3} \text{AppellF1}\left[-\frac{4}{3},-\frac{2}{3},1,-\frac{1}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{4 a d x^4 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 612: Result valid but suboptimal antiderivative.

$$\int \frac{x^6}{\left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 154 leaves, 4 steps):

$$-\frac{1}{3} x \left(1-x^3\right)^{2/3}+\frac{2 \text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6 \times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2 \times 2^{1/3}}-\frac{1}{3} \text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 3, 226 leaves, 15 steps):

$$-\frac{1}{3}x\left(1-x^3\right)^{2/3}+\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+$$

$$\frac{1}{9}\text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{2}{9}\text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 613: Result valid but suboptimal antiderivative.

$$\int\frac{x^3}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 135 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}+\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 3, 207 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{1}{6}\text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]+\frac{1}{3}\text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]+\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 614: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 88 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 615: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3 \left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 105 leaves, 3 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{2 x^2}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 3, 139 leaves, 8 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{2 x^2}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 616: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^6 \left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{5 x^5}+\frac{\left(1-x^3\right)^{2/3}}{5 x^2}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 3, 140 leaves, 9 steps):

$$-\frac{\left(1-x^3\right)^{5/3}}{5 x^5}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 617: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^9 \left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 141 leaves, 5 steps):

$$-\frac{(1-x^3)^{2/3}}{8 x^8}+\frac{(1-x^3)^{2/3}}{20 x^5}-\frac{17 (1-x^3)^{2/3}}{40 x^2}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3} x-(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 3, 175 leaves, 9 steps) :

$$-\frac{(1-x^3)^{2/3}}{2 x^2}-\frac{(1-x^3)^{5/3}}{5 x^5}-\frac{(1-x^3)^{8/3}}{8 x^8}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{(1-x^3)^{2/3}}-\frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 618: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{(1-x^3)^{1/3} (1+x^3)} \, dx$$

Optimal (type 5, 271 leaves, 12 steps) :

$$-\frac{1}{4} x^2 (1-x^3)^{2/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{1}{4} x^2 \,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$

$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step) :

$$\frac{1}{8} x^8 \,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},1,\frac{11}{3},x^3,-x^3\right]$$

Problem 619: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{(1-x^3)^{1/3} (1+x^3)} \, dx$$

Optimal (type 5, 254 leaves, 10 steps) :

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-$$
$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{5}x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]$$

Problem 620: Result unnecessarily involves higher level functions.

$$\int\frac{x}{(1-x^3)^{1/3}(1+x^3)}\,dx$$

Optimal (type 3, 233 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{2}x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]$$

Problem 621: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^2(1-x^3)^{1/3}(1+x^3)}\,dx$$

Optimal (type 5, 270 leaves, 12 steps):

$$-\frac{(1-x^3)^{2/3}}{x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-$$
$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 24 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{1}{3},\frac{1}{3},1,\frac{2}{3},x^3,-x^3\right]}{x}$$

Problem 622: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 289 leaves, 14 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{4x^4}+\frac{\left(1-x^3\right)^{2/3}}{2x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$
$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{4}{3},\frac{1}{3},1,-\frac{1}{3},x^3,-x^3\right]}{4x^4}$$

Problem 629: Result valid but suboptimal antiderivative.

$$\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 160 leaves, 5 steps):

$$-\frac{1}{3}x^2\left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}+\frac{1}{6}\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 3, 228 leaves, 14 steps):

$$-\frac{1}{3}x^2\left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-$$
$$\frac{1}{18}\text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]+\frac{1}{9}\text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]+\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}$$

Problem 630: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 139 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{1}{2} \text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 3, 207 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{1}{6} \text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{1}{3} \text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}$$

Problem 631: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 88 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}$$

Problem 632: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 \left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 103 leaves, 2 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[-2^{1/3}\,x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 3, 137 leaves, 8 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}\,x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}$$

Problem 633: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{4\,x^4}+\frac{\left(1-x^3\right)^{1/3}}{4\,x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[-2^{1/3}\,x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 3, 140 leaves, 9 steps):

$$-\frac{\left(1-x^3\right)^{4/3}}{4\,x^4}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}$$

Problem 634: Result unnecessarily involves higher level functions.

$$\int\frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 291 leaves, 15 steps):

$$-\frac{1}{2}\,x\left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{7} x^7 \text{AppellF1}\left[\frac{7}{3}, \frac{2}{3}, 1, \frac{10}{3}, x^3, -x^3\right]$$

Problem 635: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 294 leaves, 18 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3}\sqrt{3}} + \frac{1}{2} x \,\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, x^3\right] - \frac{\text{Log}\left[2^{2/3} - \frac{1-x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}} + \frac{\text{Log}\left[2\times 2^{1/3} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{4} x^4 \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 1, \frac{7}{3}, x^3, -x^3\right]$$

Problem 636: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 293 leaves, 16 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3}\sqrt{3}} + \frac{1}{2} x \,\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, x^3\right] + \frac{\text{Log}\left[2^{2/3} - \frac{1-x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}} - \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}} - \frac{\text{Log}\left[2\times 2^{1/3} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 21 leaves, 1 step):

$$x \,\text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, x^3, -x^3\right]$$

Problem 637: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 294 leaves, 16 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{2 x^2}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3} \sqrt{3}}-\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{2}{3},\frac{2}{3},1,\frac{1}{3},x^3,-x^3\right]}{2 x^2}$$

Problem 645: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 174 leaves, 5 steps):

$$\frac{x^4}{2 \left(1-x^3\right)^{1/3}}+\frac{5}{6} x \left(1-x^3\right)^{2/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}-\frac{1}{6} \text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{10} x^{10} \text{AppellF1}\left[\frac{10}{3},\frac{4}{3},1,\frac{13}{3},x^3,-x^3\right]$$

Problem 646: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{x}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{7}x^7\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},x^3,-x^3\right]$$

Problem 647: Result unnecessarily involves higher level functions.

$$\int\frac{x^3}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 106 leaves, 2 steps):

$$\frac{x}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 5, 38 leaves, 1 step):

$$\frac{x^4\,\text{Hypergeometric2F1}\left[\frac{4}{3},\frac{4}{3},\frac{7}{3},\frac{2x^3}{1+x^3}\right]}{4\left(1+x^3\right)^{4/3}}$$

Problem 648: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 106 leaves, 2 steps):

$$\frac{x}{2\left(1-x^3\right)^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 3, 140 leaves, 8 steps):

$$\frac{x}{2\left(1-x^3\right)^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}$$

Problem 649: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$\frac{1}{2 x^2 \left(1-x^3\right)^{1/3}}-\frac{\left(1-x^3\right)^{2/3}}{x^2}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 5, 204 leaves, 1 step):

$$-\frac{1}{14 x^5 \left(1-x^3\right)^{7/3}}\left(14+56 x^3-91 x^6-42 x^9+63 x^{12}-7 \left(1-x^3\right)^2 \left(2+12 x^3+9 x^6\right) \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2 x^3}{1-x^3}\right]-\right.$$
$$30 x^6 \text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2 x^3}{1-x^3}\right]-84 x^9 \text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2 x^3}{1-x^3}\right]-$$
$$\left.54 x^{12} \text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2 x^3}{1-x^3}\right]-18 x^6 \left(1+x^3\right)^2 \text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},-\frac{2 x^3}{1-x^3}\right]\right)$$

Problem 650: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{1}{2 x^5 \left(1-x^3\right)^{1/3}}-\frac{7 \left(1-x^3\right)^{2/3}}{10 x^5}-\frac{4 \left(1-x^3\right)^{2/3}}{5 x^2}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 5, 397 leaves, 1 step):

$$-\frac{1}{70\,x^8\,\left(1-x^3\right)^{7/3}}\left(28-182\,x^3-476\,x^6+819\,x^9+378\,x^{12}-567\,x^{15}-28\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2\,x^3}{1-x^3}\right]+182\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2\,x^3}{1-x^3}\right]+\right.$$
$$476\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2\,x^3}{1-x^3}\right]-819\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2\,x^3}{1-x^3}\right]-378\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2\,x^3}{1-x^3}\right]+$$
$$567\,x^{15}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2\,x^3}{1-x^3}\right]-36\,x^6\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2\,x^3}{1-x^3}\right]+$$
$$342\,x^9\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2\,x^3}{1-x^3}\right]+972\,x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2\,x^3}{1-x^3}\right]+$$
$$594\,x^{15}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2\,x^3}{1-x^3}\right]+54\,x^6\,\left(1+x^3\right)^2\,\left(1+6\,x^3\right)\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},-\frac{2\,x^3}{1-x^3}\right]+$$
$$\left.54\,x^6\,\left(1+x^3\right)^3\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},-\frac{2\,x^3}{1-x^3}\right]\right)$$

Problem 651: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^9\,\left(1-x^3\right)^{4/3}\,\left(1+x^3\right)}\,dx$$

Optimal (type 3, 162 leaves, 6 steps):

$$\frac{1}{2\,x^8\,\left(1-x^3\right)^{1/3}}-\frac{5\,\left(1-x^3\right)^{2/3}}{8\,x^8}-\frac{13\,\left(1-x^3\right)^{2/3}}{20\,x^5}-\frac{49\,\left(1-x^3\right)^{2/3}}{40\,x^2}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\,\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}\,x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 5, 612 leaves, 1 step):

$$-\frac{1}{280 x^{11} \left(1-x^3\right)^{4/3} \left(-1+x^3\right)}\Big(-70+308 x^3-1162 x^6-2856 x^9+4914 x^{12}+2268 x^{15}-3402 x^{18}+$$
$$70\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 x^3}{-1+x^3}\right]-308 x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 x^3}{-1+x^3}\right]+1162 x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 x^3}{-1+x^3}\right]+$$
$$2856 x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 x^3}{-1+x^3}\right]-4914 x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 x^3}{-1+x^3}\right]-$$
$$2268 x^{15}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 x^3}{-1+x^3}\right]+3402 x^{18}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 x^3}{-1+x^3}\right]+$$
$$66 x^6\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2 x^3}{-1+x^3}\right]-312 x^9\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2 x^3}{-1+x^3}\right]+$$
$$2268 x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2 x^3}{-1+x^3}\right]+6696 x^{15}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2 x^3}{-1+x^3}\right]+$$
$$4050 x^{18}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2 x^3}{-1+x^3}\right]+27 x^6 \left(1+x^3\right)^2 \left(-7+18 x^3+105 x^6\right) \text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{2 x^3}{-1+x^3}\right]+$$
$$54 x^6 \left(1+x^3\right)^3 \left(-1+15 x^3\right) \text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{2 x^3}{-1+x^3}\right]+$$
$$81 x^6\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2 x^3}{-1+x^3}\right]+324 x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2 x^3}{-1+x^3}\right]+$$
$$486 x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2 x^3}{-1+x^3}\right]+$$
$$324 x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2 x^3}{-1+x^3}\right]+81 x^{18}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2 x^3}{-1+x^3}\right]\Big)$$

Problem 652: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{\left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx$$

Optimal (type 5, 292 leaves, 13 steps):

$$\frac{x^5}{2 \left(1-x^3\right)^{1/3}}+\frac{3}{4} x^2 \left(1-x^3\right)^{2/3}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\sqrt{3}}-\frac{1}{2} x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-$$
$$\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{24\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{11} x^{11} \text{AppellF1}\left[\frac{11}{3}, \frac{4}{3}, 1, \frac{14}{3}, x^3, -x^3\right]$$

Problem 653: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 274 leaves, 12 steps):

$$\frac{x^2}{2 \left(1-x^3\right)^{1/3}} + \frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3} \sqrt{3}} - \frac{3}{4} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] + \frac{\text{Log}\left[(1-x) (1+x)^2\right]}{24\times 2^{1/3}} + \frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}} - \frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}} - \frac{\text{Log}\left[-1+x+2^{2/3} \left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{8} x^8 \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, x^3, -x^3\right]$$

Problem 654: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 274 leaves, 12 steps):

$$\frac{x^2}{2 \left(1-x^3\right)^{1/3}} - \frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3} \sqrt{3}} - \frac{1}{4} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] - \frac{\text{Log}\left[(1-x) (1+x)^2\right]}{24\times 2^{1/3}} - \frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}} + \frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}} + \frac{\text{Log}\left[-1+x+2^{2/3} \left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{5} x^5 \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, x^3, -x^3\right]$$

Problem 655: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 274 leaves, 11 steps):

$$\frac{x^2}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\sqrt{3}}-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$
$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{2}x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{4}{3},1,\frac{5}{3},x^3,-x^3\right]$$

Problem 656: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 292 leaves, 13 steps):

$$\frac{1}{2x\left(1-x^3\right)^{1/3}}-\frac{3\left(1-x^3\right)^{2/3}}{2x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\sqrt{3}}-\frac{3}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-$$
$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 24 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{1}{3},\frac{4}{3},1,\frac{2}{3},x^3,-x^3\right]}{x}$$

Problem 657: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 308 leaves, 14 steps):

$$\frac{1}{2 x^4 \left(1-x^3\right)^{1/3}}-\frac{3 \left(1-x^3\right)^{2/3}}{4 x^4}-\frac{\left(1-x^3\right)^{2/3}}{x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3} \sqrt{3}}-\frac{1}{2} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$
$$\frac{\text{Log}\left[(1-x) (1+x)^2\right]}{24\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3} \left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{4}{3},\frac{4}{3},1,-\frac{1}{3},x^3,-x^3\right]}{4 x^4}$$

Problem 665: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 \left(a+b x^3\right)^{1/3}}{c+d x^3} \, dx$$

Optimal (type 3, 336 leaves, 6 steps):

$$-\frac{(6 b c-a d) x^2 \left(a+b x^3\right)^{1/3}}{18 b d^2}+\frac{x^5 \left(a+b x^3\right)^{1/3}}{6 d}-\frac{\left(9 b^2 c^2-3 a b c d-a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{5/3} d^3}+\frac{c^{5/3} (b c-a d)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^3}-$$
$$\frac{c^{5/3} (b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 d^3}-\frac{\left(9 b^2 c^2-3 a b c d-a^2 d^2\right) \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{18 b^{5/3} d^3}+\frac{c^{5/3} (b c-a d)^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^3}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^8 \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{8}{3},-\frac{1}{3},1,\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{8 c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 666: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \left(a+b x^3\right)^{1/3}}{c+d x^3} \, dx$$

Optimal (type 3, 276 leaves, 5 steps):

$$\frac{x^2 \left(a+b x^3\right)^{1/3}}{3 d}+\frac{(3 b c-a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3} d^2}-\frac{c^{2/3} (b c-a d)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2}+$$
$$\frac{c^{2/3} (b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 d^2}+\frac{(3 b c-a d) \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 b^{2/3} d^2}-\frac{c^{2/3} (b c-a d)^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^2}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^5 \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{5}{3},-\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{5 c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 667: Result unnecessarily involves higher level functions.

$$\int \frac{x \left(a+b x^3\right)^{1/3}}{c+d x^3} \, dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$-\frac{b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d}+\frac{(b c-a d)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{1/3} d}-$$
$$\frac{(b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 c^{1/3} d}-\frac{b^{1/3} \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 d}+\frac{(b c-a d)^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{1/3} d}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^2 \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 668: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^2 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 168 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{c x}-\frac{(b c-a d)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{4/3}}+\frac{(b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 c^{4/3}}-\frac{(b c-a d)^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{4/3}}$$

Result (type 5, 87 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{1/3} \left(1+\frac{d x^3}{c}\right)^{1/3} \text{Hypergeometric2F1}\left[-\frac{1}{3},-\frac{1}{3},\frac{2}{3},-\frac{c \left(\frac{b x^3}{a}-\frac{d x^3}{c}\right)}{c+d x^3}\right]}{c x \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 669: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^5 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 204 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{4 c x^4}-\frac{(b c-4 a d) \left(a+b x^3\right)^{1/3}}{4 a c^2 x}+\frac{d (b c-a d)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{7/3}}-$$
$$\frac{d (b c-a d)^{1/3} \text{Log}\left[c+d x^3\right]}{6 c^{7/3}}+\frac{d (b c-a d)^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{7/3}}$$

Result (type 5, 145 leaves, 2 steps):

$$-\frac{1}{8 c^3 x^4 \left(a+b x^3\right)^{2/3}}\left(2 c \left(a+b x^3\right) \left(c-3 d x^3\right)-\right.$$
$$\left.(b c-a d) x^3 \left(c-3 d x^3\right) \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]+3 (b c-a d) x^3 \left(c+d x^3\right) \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]\right)$$

Problem 670: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^8 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 258 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{7 c x^7}-\frac{\left(b c-7 a d\right) \left(a+b x^3\right)^{1/3}}{28 a c^2 x^4}+\frac{\left(3 b^2 c^2+7 a b c d-28 a^2 d^2\right) \left(a+b x^3\right)^{1/3}}{28 a^2 c^3 x}-\frac{d^2 \left(b c-a d\right)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 \left(b c-a d\right)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{10/3}}+\frac{d^2 \left(b c-a d\right)^{1/3} \text{Log}\left[c+d x^3\right]}{6 c^{10/3}}-\frac{d^2 \left(b c-a d\right)^{1/3} \text{Log}\left[\frac{\left(b c-a d\right)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{10/3}}$$

Result (type 5, 451 leaves, 2 steps):

$$-\frac{1}{56 c^4 x^7 \left(a+b x^3\right)^{2/3}}\left(8 a c^3+8 b c^3 x^3-12 a c^2 d x^3-12 b c^2 d x^6+36 a c d^2 x^6+36 b c d^2 x^9-2 \left(b c-a d\right) x^3 \left(2 c^2-3 c d x^3+9 d^2 x^6\right) \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+15 b c^3 x^3 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-15 a c^2 d x^3 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-12 b c^2 d x^6 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+12 a c d^2 x^6 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-27 b c d^2 x^9 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+27 a d^3 x^9 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-9 \left(b c-a d\right) x^3 \left(c+d x^3\right)^2 \text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{5}{3}\right\},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]\right)$$

Problem 671: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^{11} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{10 c x^{10}}-\frac{\left(b c-10 a d\right)\left(a+b x^3\right)^{1/3}}{70 a c^2 x^7}+\frac{\left(3 b^2 c^2+5 a b c d-35 a^2 d^2\right)\left(a+b x^3\right)^{1/3}}{140 a^2 c^3 x^4}-\frac{\left(9 b^3 c^3+15 a b^2 c^2 d+35 a^2 b c d^2-140 a^3 d^3\right)\left(a+b x^3\right)^{1/3}}{140 a^3 c^4 x}+$$

$$\frac{d^3\left(b c-a d\right)^{1/3}\text{ArcTan}\left[\frac{1+\frac{2\left(b c-a d\right)^{1/3}x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{13/3}}-\frac{d^3\left(b c-a d\right)^{1/3}\text{Log}\left[c+d x^3\right]}{6 c^{13/3}}+\frac{d^3\left(b c-a d\right)^{1/3}\text{Log}\left[\frac{\left(b c-a d\right)^{1/3}x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{13/3}}$$

Result (type 5, 905 leaves, 2 steps):

$$-\frac{1}{560 c^5 x^{10}\left(a+b x^3\right)^{2/3}}\Bigg(56 a c^4+56 b c^4 x^3-72 a c^3 d x^3-72 b c^3 d x^6+108 a c^2 d^2 x^6+108 b c^2 d^2 x^9-324 a c d^3 x^9-324 b c d^3 x^{12}-$$

$$28 b c^4 x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]+28 a c^3 d x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]+$$

$$36 b c^3 d x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]-36 a c^2 d^2 x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]-$$

$$54 b c^2 d^2 x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]+54 a c d^3 x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]+$$

$$162 b c d^3 x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]-162 a d^4 x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]+$$

$$117 b c^4 x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]-117 a c^3 d x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]-$$

$$99 b c^3 d x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]+99 a c^2 d^2 x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]+$$

$$81 b c^2 d^2 x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]-81 a c d^3 x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]+$$

$$297 b c d^3 x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]-297 a d^4 x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]-$$

$$54\left(b c-a d\right)x^3\left(2 c-3 d x^3\right)\left(c+d x^3\right)^2\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{5}{3}\right\},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]+$$

$$27\left(b c-a d\right)x^3\left(c+d x^3\right)^3\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{5}{3}\right\},\frac{\left(b c-a d\right)x^3}{c\left(a+b x^3\right)}\right]\Bigg)$$

Problem 684: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

Optimal (type 3, 334 leaves, 5 steps):

$$-\frac{(3 b c-a d) x \left(a+b x^3\right)^{2/3}}{9 b d^2}+\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 d}+\frac{\left(9 b^2 c^2-6 a b c d-a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{4/3} d^3}-\frac{c^{4/3} (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^3}-\frac{c^{4/3} (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 d^3}+\frac{c^{4/3} (b c-a d)^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^3}-\frac{\left(9 b^2 c^2-6 a b c d-a^2 d^2\right) \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{18 b^{4/3} d^3}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^7 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{7}{3},-\frac{2}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{7 c \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 685: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

Optimal (type 3, 272 leaves, 4 steps):

$$\frac{x \left(a+b x^3\right)^{2/3}}{3 d}-\frac{(3 b c-2 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{1/3} d^2}+\frac{c^{1/3} (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2}+\frac{c^{1/3} (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 d^2}-\frac{c^{1/3} (b c-a d)^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^2}+\frac{(3 b c-2 a d) \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 b^{1/3} d^2}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^4 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{4}{3},-\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 c \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 686: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{b^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d}-\frac{(b c-a d)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} d}-\frac{(b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 c^{2/3} d}+\frac{(b c-a d)^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} d}-\frac{b^{2/3} \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 d}$$

Result (type 6, 59 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 687: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^3 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 169 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{2 c x^2}+\frac{(b c-a d)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{5/3}}+\frac{(b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 c^{5/3}}-\frac{(b c-a d)^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{5/3}}$$

Result (type 5, 89 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{2/3} \left(1+\frac{d x^3}{c}\right)^{2/3} \text{Hypergeometric2F1}\left[-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\frac{c \left(\frac{b x^3}{a}-\frac{d x^3}{c}\right)}{c+d x^3}\right]}{2 c x^2 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 688: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^6 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 206 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{5 c x^5}-\frac{\left(2 b c-5 a d\right)\left(a+b x^3\right)^{2/3}}{10 a c^2 x^2}-\frac{d \left(b c-a d\right)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 \left(b c-a d\right)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{8/3}}-\frac{d \left(b c-a d\right)^{2/3} \text{Log}\left[c+d x^3\right]}{6 c^{8/3}}+\frac{d \left(b c-a d\right)^{2/3} \text{Log}\left[\frac{\left(b c-a d\right)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{8/3}}$$

Result (type 5, 148 leaves, 2 steps):

$$-\frac{1}{10 c^3 x^5 \left(a+b x^3\right)^{1/3}}\left(c \left(a+b x^3\right) \left(2 c-3 d x^3\right)-2 \left(b c-a d\right) x^3 \left(2 c-3 d x^3\right) \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+6 \left(b c-a d\right) x^3 \left(c+d x^3\right) \text{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]\right)$$

Problem 689: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^9 \left(c+d x^3\right)}\, dx$$

Optimal (type 3, 257 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{8 c x^8}-\frac{\left(b c-4 a d\right)\left(a+b x^3\right)^{2/3}}{20 a c^2 x^5}+\frac{\left(3 b^2 c^2+8 a b c d-20 a^2 d^2\right)\left(a+b x^3\right)^{2/3}}{40 a^2 c^3 x^2}+\frac{d^2 \left(b c-a d\right)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 \left(b c-a d\right)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{11/3}}+\frac{d^2 \left(b c-a d\right)^{2/3} \text{Log}\left[c+d x^3\right]}{6 c^{11/3}}-\frac{d^2 \left(b c-a d\right)^{2/3} \text{Log}\left[\frac{\left(b c-a d\right)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{11/3}}$$

Result (type 5, 451 leaves, 2 steps):

$$-\frac{1}{40\,c^4\,x^8\,\left(a+b\,x^3\right)^{1/3}}\left(5\,a\,c^3+5\,b\,c^3\,x^3-6\,a\,c^2\,d\,x^3-6\,b\,c^2\,d\,x^6+9\,a\,c\,d^2\,x^6+9\,b\,c\,d^2\,x^9-\right.$$
$$2\,(b\,c-a\,d)\,x^3\,\left(5\,c^2-6\,c\,d\,x^3+9\,d^2\,x^6\right)\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+21\,b\,c^3\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$
$$21\,a\,c^2\,d\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-6\,b\,c^2\,d\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$
$$6\,a\,c\,d^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-27\,b\,c\,d^2\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$
$$\left.27\,a\,d^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-9\,(b\,c-a\,d)\,x^3\,\left(c+d\,x^3\right)^2\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{3},2,2\right\},\left\{1,\frac{4}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,\left(a+b\,x^3\right)}\right]\right)$$

Problem 690: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\left(a+b\,x^3\right)^{2/3}}{x^{12}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 320 leaves, 6 steps):

$$-\frac{\left(a+b\,x^3\right)^{2/3}}{11\,c\,x^{11}}-\frac{(2\,b\,c-11\,a\,d)\,\left(a+b\,x^3\right)^{2/3}}{88\,a\,c^2\,x^8}+\frac{\left(6\,b^2\,c^2+11\,a\,b\,c\,d-44\,a^2\,d^2\right)\,\left(a+b\,x^3\right)^{2/3}}{220\,a^2\,c^3\,x^5}-$$
$$\frac{\left(18\,b^3\,c^3+33\,a\,b^2\,c^2\,d+88\,a^2\,b\,c\,d^2-220\,a^3\,d^3\right)\,\left(a+b\,x^3\right)^{2/3}}{440\,a^3\,c^4\,x^2}-\frac{d^3\,(b\,c-a\,d)^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{14/3}}-$$
$$\frac{d^3\,(b\,c-a\,d)^{2/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{14/3}}+\frac{d^3\,(b\,c-a\,d)^{2/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{14/3}}$$

Result (type 5, 819 leaves, 2 steps):

$$-\frac{1}{440\,c^5\,x^{11}\,(a+b\,x^3)^{1/3}}\Big(40\,a\,c^4+40\,b\,c^4\,x^3-45\,a\,c^3\,d\,x^3-45\,b\,c^3\,d\,x^6+54\,a\,c^2\,d^2\,x^6+54\,b\,c^2\,d^2\,x^9-81\,a\,c\,d^3\,x^9-81\,b\,c\,d^3\,x^{12}-$$
$$80\,b\,c^4\,x^3\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+80\,a\,c^3\,d\,x^3\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$90\,b\,c^3\,d\,x^6\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-90\,a\,c^2\,d^2\,x^6\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$108\,b\,c^2\,d^2\,x^9\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+108\,a\,c\,d^3\,x^9\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$162\,b\,c\,d^3\,x^{12}\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-162\,a\,d^4\,x^{12}\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$198\,b\,c^4\,x^3\,\text{Hypergeometric2F1}\Big[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-198\,a\,c^3\,d\,x^3\,\text{Hypergeometric2F1}\Big[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$99\,b\,c^3\,d\,x^6\,\text{Hypergeometric2F1}\Big[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+99\,a\,c^2\,d^2\,x^6\,\text{Hypergeometric2F1}\Big[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$297\,b\,c\,d^3\,x^{12}\,\text{Hypergeometric2F1}\Big[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-297\,a\,d^4\,x^{12}\,\text{Hypergeometric2F1}\Big[\frac{1}{3},2,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$27\,(b\,c-a\,d)\,x^3\,(5\,c-6\,d\,x^3)\,(c+d\,x^3)^2\,\text{HypergeometricPFQ}\Big[\Big\{\frac{1}{3},2,2\Big\},\Big\{1,\frac{4}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$27\,(b\,c-a\,d)\,x^3\,(c+d\,x^3)^3\,\text{HypergeometricPFQ}\Big[\Big\{\frac{1}{3},2,2,2\Big\},\Big\{1,1,\frac{4}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]\Big)$$

Problem 702: Result unnecessarily involves higher level functions.

$$\int\frac{x^4\,(a+b\,x^3)^{4/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 334 leaves, 6 steps):

$$-\frac{(6\,b\,c-7\,a\,d)\,x^2\,(a+b\,x^3)^{1/3}}{18\,d^2}+\frac{b\,x^5\,(a+b\,x^3)^{1/3}}{6\,d}-\frac{(9\,b^2\,c^2-12\,a\,b\,c\,d+2\,a^2\,d^2)\,\text{ArcTan}\Big[\frac{1+\frac{2\,b^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}}\Big]}{9\,\sqrt{3}\,b^{2/3}\,d^3}+\frac{c^{2/3}\,(b\,c-a\,d)^{4/3}\,\text{ArcTan}\Big[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\Big]}{\sqrt{3}\,d^3}-$$
$$\frac{c^{2/3}\,(b\,c-a\,d)^{4/3}\,\text{Log}[c+d\,x^3]}{6\,d^3}-\frac{(9\,b^2\,c^2-12\,a\,b\,c\,d+2\,a^2\,d^2)\,\text{Log}\big[b^{1/3}\,x-(a+b\,x^3)^{1/3}\big]}{18\,b^{2/3}\,d^3}+\frac{c^{2/3}\,(b\,c-a\,d)^{4/3}\,\text{Log}\Big[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\Big]}{2\,d^3}$$

Result (type 6, 65 leaves, 2 steps):

$$\frac{a x^5 \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{5}{3},-\frac{4}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{5 c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 703: Result unnecessarily involves higher level functions.

$$\int \frac{x \left(a+b x^3\right)^{4/3}}{c+d x^3} \, dx$$

Optimal (type 3, 277 leaves, 5 steps):

$$\frac{b x^2 \left(a+b x^3\right)^{1/3}}{3 d}+\frac{b^{1/3} (3 b c-4 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} d^2}-\frac{(b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{1/3} d^2}+$$
$$\frac{(b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{1/3} d^2}+\frac{b^{1/3} (3 b c-4 a d) \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 d^2}-\frac{(b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{1/3} d^2}$$

Result (type 6, 65 leaves, 2 steps):

$$\frac{a x^2 \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{2}{3},-\frac{4}{3},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 704: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^2 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 254 leaves, 5 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3}}{c x}-\frac{b^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d}+\frac{(b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{4/3} d}-$$
$$\frac{(b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{4/3} d}-\frac{b^{4/3} \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 d}+\frac{(b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{4/3} d}$$

Result (type 6, 63 leaves, 2 steps):

$$-\frac{a\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[-\frac{1}{3},-\frac{4}{3},1,\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c\, x\left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 705: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^5\left(c+d x^3\right)}\,dx$$

Optimal (type 3, 201 leaves, 4 steps):

$$-\frac{a\left(a+b x^3\right)^{1/3}}{4 c x^4}-\frac{(5 b c-4 a d)\left(a+b x^3\right)^{1/3}}{4 c^2 x}-\frac{(b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{7/3}}+\frac{(b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{7/3}}-\frac{(b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{7/3}}$$

Result (type 5, 90 leaves, 2 steps):

$$-\frac{a\left(a+b x^3\right)^{1/3}\left(1+\frac{d x^3}{c}\right)^{4/3} \text{Hypergeometric2F1}\left[-\frac{4}{3},-\frac{4}{3},-\frac{1}{3},-\frac{c\left(\frac{b x^3}{a}-\frac{d x^3}{c}\right)}{c+d x^3}\right]}{4 c x^4\left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 706: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^8\left(c+d x^3\right)}\,dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$-\frac{a\left(a+b x^3\right)^{1/3}}{7 c x^7}-\frac{(8 b c-7 a d)\left(a+b x^3\right)^{1/3}}{28 c^2 x^4}-\frac{\left(4 b^2 c^2-35 a b c d+28 a^2 d^2\right)\left(a+b x^3\right)^{1/3}}{28 a c^3 x}+\frac{d (b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{10/3}}-\frac{d (b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{10/3}}+\frac{d (b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{10/3}}$$

Result (type 5, 169 leaves, 2 steps):

$$\frac{1}{28\,c^4\,x^7\,\left(a+b\,x^3\right)^{2/3}}\left(12\,c\,\left(b\,c-a\,d\right)\,x^3\,\left(a+b\,x^3\right)\,\left(c+d\,x^3\right)\,\text{Hypergeometric2F1}\left[-\frac{1}{3},\,2,\,\frac{2}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-\left(4\,c-3\,d\,x^3\right)\,\left(c\,\left(a+b\,x^3\right)\,\left(5\,b\,c\,x^3+a\,\left(c-4\,d\,x^3\right)\right)-2\,\left(b\,c-a\,d\right)^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,1,\,\frac{5}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]\right)\right)$$

Problem 707: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{4/3}}{x^{11}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$-\frac{a\,\left(a+b\,x^3\right)^{1/3}}{10\,c\,x^{10}}-\frac{\left(11\,b\,c-10\,a\,d\right)\,\left(a+b\,x^3\right)^{1/3}}{70\,c^2\,x^7}-\frac{\left(2\,b^2\,c^2-40\,a\,b\,c\,d+35\,a^2\,d^2\right)\,\left(a+b\,x^3\right)^{1/3}}{140\,a\,c^3\,x^4}+\frac{\left(6\,b^3\,c^3+20\,a\,b^2\,c^2\,d-175\,a^2\,b\,c\,d^2+140\,a^3\,d^3\right)\,\left(a+b\,x^3\right)^{1/3}}{140\,a^2\,c^4\,x}-\frac{d^2\,\left(b\,c-a\,d\right)^{4/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{13/3}}+\frac{d^2\,\left(b\,c-a\,d\right)^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{13/3}}-\frac{d^2\,\left(b\,c-a\,d\right)^{4/3}\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{13/3}}$$

Result (type 5, 260 leaves, 2 steps):

$$\frac{1}{140\,c^5\,x^{10}\,\left(a+b\,x^3\right)^{2/3}}\left(6\,c\,\left(b\,c-a\,d\right)\,x^3\,\left(a+b\,x^3\right)\,\left(11\,c^2+2\,c\,d\,x^3-9\,d^2\,x^6\right)\,\text{Hypergeometric2F1}\left[-\frac{1}{3},\,2,\,\frac{2}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-\left(14\,c^2-12\,c\,d\,x^3+9\,d^2\,x^6\right)\,\left(c\,\left(a+b\,x^3\right)\,\left(5\,b\,c\,x^3+a\,\left(c-4\,d\,x^3\right)\right)-2\,\left(b\,c-a\,d\right)^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,1,\,\frac{5}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]\right)-18\,c\,\left(b\,c-a\,d\right)\,x^3\,\left(a+b\,x^3\right)\,\left(c+d\,x^3\right)^2\,\text{HypergeometricPFQ}\left[\left\{-\frac{1}{3},\,2,\,2\right\},\,\left\{\frac{2}{3},\,1\right\},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]\right)$$

Problem 708: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\left(a+b\,x^3\right)^{4/3}}{x^{14}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 392 leaves, 7 steps):

$$-\frac{a\left(a+b x^3\right)^{1/3}}{13 c x^{13}}-\frac{(14 b c-13 a d)\left(a+b x^3\right)^{1/3}}{130 c^2 x^{10}}-\frac{\left(4 b^2 c^2-143 a b c d+130 a^2 d^2\right)\left(a+b x^3\right)^{1/3}}{910 a c^3 x^7}+$$
$$\frac{\left(12 b^3 c^3+26 a b^2 c^2 d-520 a^2 b c d^2+455 a^3 d^3\right)\left(a+b x^3\right)^{1/3}}{1820 a^2 c^4 x^4}-\frac{\left(36 b^4 c^4+78 a b^3 c^3 d+260 a^2 b^2 c^2 d^2-2275 a^3 b c d^3+1820 a^4 d^4\right)\left(a+b x^3\right)^{1/3}}{1820 a^3 c^5 x}+$$
$$\frac{d^3 (b c-a d)^{4/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{16/3}}-\frac{d^3 (b c-a d)^{4/3} \operatorname{Log}\left[c+d x^3\right]}{6 c^{16/3}}+\frac{d^3 (b c-a d)^{4/3} \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{16/3}}$$

Result (type 5, 1446 leaves, 2 steps):

$$-\frac{1}{1820 c^6 x^{13}\left(a+b x^3\right)^{2/3}}$$
$$\Big(140 a^2 c^5+840 a b c^5 x^3-686 a^2 c^4 d x^3+700 b^2 c^5 x^6-1316 a b c^4 d x^6+612 a^2 c^3 d^2 x^6-630 b^2 c^4 d x^9+1152 a b c^3 d^2 x^9-513 a^2 c^2 d^3 x^9+$$
$$540 b^2 c^3 d^2 x^{12}-918 a b c^2 d^3 x^{12}+324 a^2 c d^4 x^{12}-405 b^2 c^2 d^3 x^{15}+324 a b c d^4 x^{15}-828 a b c^5 x^3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+$$
$$828 a^2 c^4 d x^3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]-828 b^2 c^5 x^6 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+$$
$$918 a b c^4 d x^6 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]-90 a^2 c^3 d^2 x^6 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+$$
$$90 b^2 c^4 d x^9 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+234 a b c^3 d^2 x^9 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]-$$
$$324 a^2 c^2 d^3 x^9 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+324 b^2 c^3 d^2 x^{12} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]-$$
$$918 a b c^2 d^3 x^{12} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+594 a^2 c d^4 x^{12} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]-$$
$$594 b^2 c^2 d^3 x^{15} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+594 a b c d^4 x^{15} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]-$$
$$280 b^2 c^5 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+560 a b c^4 d x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]-$$
$$280 a^2 c^3 d^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+252 b^2 c^4 d x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]-$$
$$504 a b c^3 d^2 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]+252 a^2 c^2 d^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c-a d) x^3}{c\left(a+b x^3\right)}\right]-$$

$$216\, b^2 c^3 d^2 x^{12}\, \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] + 432\, a\, b\, c^2 d^3 x^{12}\, \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$$

$$216\, a^2 c\, d^4 x^{12}\, \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] + 162\, b^2 c^2 d^3 x^{15}\, \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$$

$$324\, a\, b\, c\, d^4 x^{15}\, \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] + 162\, a^2 d^5 x^{15}\, \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$$

$$54\, c\, (b c - a d)\, x^3\, (a + b x^3)\, (7 c - 6 d x^3)\, (c + d x^3)^2\, \text{HypergeometricPFQ}\left[\left\{-\frac{1}{3}, 2, 2\right\}, \left\{\frac{2}{3}, 1\right\}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$$

$$54\, c\, (b c - a d)\, x^3\, (a + b x^3)\, (c + d x^3)^3\, \text{HypergeometricPFQ}\left[\left\{-\frac{1}{3}, 2, 2, 2\right\}, \left\{\frac{2}{3}, 1, 1\right\}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right]\Bigg)$$

Problem 721: Result valid but suboptimal antiderivative.

$$\int \frac{x^6}{(a + b x^3)^{1/3}\, (c + d x^3)}\, dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$\frac{x\, (a + b x^3)^{2/3}}{3\, b\, d} - \frac{(3 b c + a d)\, \text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{4/3} d^2} + \frac{c^{4/3}\, \text{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^2\, (b c - a d)^{1/3}} +$$

$$\frac{c^{4/3}\, \text{Log}\left[c + d x^3\right]}{6\, d^2\, (b c - a d)^{1/3}} - \frac{c^{4/3}\, \text{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2\, d^2\, (b c - a d)^{1/3}} + \frac{(3 b c + a d)\, \text{Log}\left[-b^{1/3} x + (a + b x^3)^{1/3}\right]}{6\, b^{4/3} d^2}$$

Result (type 3, 394 leaves, 15 steps):

$$\frac{x\, (a + b x^3)^{2/3}}{3\, b\, d} - \frac{(3 b c + a d)\, \text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{4/3} d^2} + \frac{c^{4/3}\, \text{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, d^2\, (b c - a d)^{1/3}} + \frac{(3 b c + a d)\, \text{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9\, b^{4/3} d^2} -$$

$$\frac{(3 b c + a d)\, \text{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{18\, b^{4/3} d^2} - \frac{c^{4/3}\, \text{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3\, d^2\, (b c - a d)^{1/3}} + \frac{c^{4/3}\, \text{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6\, d^2\, (b c - a d)^{1/3}}$$

Problem 722: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3} d}-\frac{c^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d\, (b c-a d)^{1/3}}-\frac{c^{1/3} \text{Log}\left[c+d x^3\right]}{6 d\, (b c-a d)^{1/3}}+\frac{c^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d\, (b c-a d)^{1/3}}-\frac{\text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 b^{1/3} d}$$

Result (type 3, 346 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3} d}-\frac{c^{1/3} \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, d\, (b c-a d)^{1/3}}-\frac{\text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3} d}+$$

$$\frac{\text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 b^{1/3} d}+\frac{c^{1/3} \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 d\, (b c-a d)^{1/3}}-\frac{c^{1/3} \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 d\, (b c-a d)^{1/3}}$$

Problem 723: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 148 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} (b c-a d)^{1/3}}+\frac{\text{Log}\left[c+d x^3\right]}{6 c^{2/3} (b c-a d)^{1/3}}-\frac{\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} (b c-a d)^{1/3}}$$

Result (type 3, 207 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{2/3} (b c-a d)^{1/3}}-\frac{\text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{2/3} (b c-a d)^{1/3}}+\frac{\text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{2/3} (b c-a d)^{1/3}}$$

Problem 724: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3 \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 176 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{2 a c x^2}-\frac{d \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{5/3} (b c-a d)^{1/3}}-\frac{d \operatorname{Log}\left[c+d x^3\right]}{6 c^{5/3} (b c-a d)^{1/3}}+\frac{d \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{5/3} (b c-a d)^{1/3}}$$

Result (type 3, 235 leaves, 8 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{2 a c x^2}-\frac{d \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{5/3} (b c-a d)^{1/3}}+\frac{d \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{5/3} (b c-a d)^{1/3}}-\frac{d \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{5/3} (b c-a d)^{1/3}}$$

Problem 725: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^6 \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 214 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{5 a c x^5}+\frac{(3 b c+5 a d) \left(a+b x^3\right)^{2/3}}{10 a^2 c^2 x^2}+\frac{d^2 \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{8/3} (b c-a d)^{1/3}}+\frac{d^2 \operatorname{Log}\left[c+d x^3\right]}{6 c^{8/3} (b c-a d)^{1/3}}-\frac{d^2 \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{8/3} (b c-a d)^{1/3}}$$

Result (type 3, 271 leaves, 9 steps):

$$\frac{(b c+a d) \left(a+b x^3\right)^{2/3}}{2 a^2 c^2 x^2}-\frac{\left(a+b x^3\right)^{5/3}}{5 a^2 c x^5}+\frac{d^2 \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{8/3} (b c-a d)^{1/3}}-\frac{d^2 \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{8/3} (b c-a d)^{1/3}}+\frac{d^2 \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{8/3} (b c-a d)^{1/3}}$$

Problem 726: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^9 \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 262 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{8 a c x^8}+\frac{(3 b c+4 a d)\left(a+b x^3\right)^{2/3}}{20 a^2 c^2 x^5}-\frac{\left(9 b^2 c^2+12 a b c d+20 a^2 d^2\right)\left(a+b x^3\right)^{2/3}}{40 a^3 c^3 x^2}-$$

$$\frac{d^3 \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{11/3} (b c-a d)^{1/3}}-\frac{d^3 \operatorname{Log}\left[c+d x^3\right]}{6 c^{11/3} (b c-a d)^{1/3}}+\frac{d^3 \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{11/3} (b c-a d)^{1/3}}$$

Result (type 3, 317 leaves, 9 steps):

$$-\frac{\left(b^2 c^2+a b c d+a^2 d^2\right)\left(a+b x^3\right)^{2/3}}{2 a^3 c^3 x^2}+\frac{(2 b c+a d)\left(a+b x^3\right)^{5/3}}{5 a^3 c^2 x^5}-\frac{\left(a+b x^3\right)^{8/3}}{8 a^3 c x^8}-$$

$$\frac{d^3 \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{11/3} (b c-a d)^{1/3}}+\frac{d^3 \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{11/3} (b c-a d)^{1/3}}-\frac{d^3 \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{11/3} (b c-a d)^{1/3}}$$

Problem 738: Result valid but suboptimal antiderivative.

$$\int \frac{x^7}{\left(a+b x^3\right)^{2/3}\left(c+d x^3\right)} \, dx$$

Optimal (type 3, 279 leaves, 5 steps):

$$\frac{x^2\left(a+b x^3\right)^{1/3}}{3 b d}+\frac{(3 b c+2 a d) \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{5/3} d^2}-\frac{c^{5/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^2 (b c-a d)^{2/3}}+$$

$$\frac{c^{5/3} \operatorname{Log}\left[c+d x^3\right]}{6 d^2 (b c-a d)^{2/3}}+\frac{(3 b c+2 a d) \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 b^{5/3} d^2}-\frac{c^{5/3} \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^2 (b c-a d)^{2/3}}$$

Result (type 3, 400 leaves, 16 steps):

$$\frac{x^2\left(a+b x^3\right)^{1/3}}{3 b d}+\frac{(3 b c+2 a d) \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{5/3} d^2}-\frac{c^{5/3} \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, d^2 (b c-a d)^{2/3}}+\frac{(3 b c+2 a d) \operatorname{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{5/3} d^2}-$$

$$\frac{(3 b c+2 a d) \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{18 b^{5/3} d^2}-\frac{c^{5/3} \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 d^2 (b c-a d)^{2/3}}+\frac{c^{5/3} \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 d^2 (b c-a d)^{2/3}}$$

Problem 739: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3} d}+\frac{c^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d (b c-a d)^{2/3}}-\frac{c^{2/3} \text{Log}\left[c+d x^3\right]}{6 d (b c-a d)^{2/3}}-\frac{\text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 b^{2/3} d}+\frac{c^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d (b c-a d)^{2/3}}$$

Result (type 3, 346 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3} d}+\frac{c^{2/3} \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, d (b c-a d)^{2/3}}-\frac{\text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{2/3} d}+$$

$$\frac{\text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 b^{2/3} d}+\frac{c^{2/3} \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 d (b c-a d)^{2/3}}-\frac{c^{2/3} \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 d (b c-a d)^{2/3}}$$

Problem 740: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 149 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{1/3} (b c-a d)^{2/3}}+\frac{\text{Log}\left[c+d x^3\right]}{6 c^{1/3} (b c-a d)^{2/3}}-\frac{\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{1/3} (b c-a d)^{2/3}}$$

Result (type 3, 208 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{1/3} (b c-a d)^{2/3}}-\frac{\text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{1/3} (b c-a d)^{2/3}}+\frac{\text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{1/3} (b c-a d)^{2/3}}$$

Problem 741: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 173 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{a c x}+\frac{d \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{4/3} (b c-a d)^{2/3}}-\frac{d \operatorname{Log}\left[c+d x^3\right]}{6 c^{4/3} (b c-a d)^{2/3}}+\frac{d \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{4/3} (b c-a d)^{2/3}}$$

Result (type 3, 232 leaves, 8 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{a c x}+\frac{d \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{4/3} (b c-a d)^{2/3}}+\frac{d \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{4/3} (b c-a d)^{2/3}}-\frac{d \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{4/3} (b c-a d)^{2/3}}$$

Problem 742: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 215 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{4 a c x^4}+\frac{(3 b c+4 a d) \left(a+b x^3\right)^{1/3}}{4 a^2 c^2 x}-\frac{d^2 \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{7/3} (b c-a d)^{2/3}}+\frac{d^2 \operatorname{Log}\left[c+d x^3\right]}{6 c^{7/3} (b c-a d)^{2/3}}-\frac{d^2 \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{7/3} (b c-a d)^{2/3}}$$

Result (type 3, 269 leaves, 9 steps):

$$\frac{(b c+a d) \left(a+b x^3\right)^{1/3}}{a^2 c^2 x}-\frac{\left(a+b x^3\right)^{4/3}}{4 a^2 c x^4}-\frac{d^2 \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{7/3} (b c-a d)^{2/3}}-\frac{d^2 \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{7/3} (b c-a d)^{2/3}}+\frac{d^2 \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{7/3} (b c-a d)^{2/3}}$$

Problem 754: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 322 leaves, 5 steps):

$$\frac{a x^4}{b (b c-a d) (a+b x^3)^{1/3}}+\frac{(b c-4 a d) x (a+b x^3)^{2/3}}{3 b^2 d (b c-a d)}-\frac{(3 b c+4 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{7/3} d^2}+$$

$$\frac{c^{7/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2 (b c-a d)^{4/3}}+\frac{c^{7/3} \text{Log}\left[c+d x^3\right]}{6 d^2 (b c-a d)^{4/3}}-\frac{c^{7/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-(a+b x^3)^{1/3}\right]}{2 d^2 (b c-a d)^{4/3}}+\frac{(3 b c+4 a d) \text{Log}\left[-b^{1/3} x+(a+b x^3)^{1/3}\right]}{6 b^{7/3} d^2}$$

Result (type 6, 67 leaves, 2 steps) :

$$\frac{x^{10} \left(1+\frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{10}{3},\frac{4}{3},1,\frac{13}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{10 a c (a+b x^3)^{1/3}}$$

Problem 755: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(a+b x^3)^{4/3} (c+d x^3)} \, dx$$

Optimal (type 3, 260 leaves, 4 steps) :

$$\frac{a x}{b (b c-a d) (a+b x^3)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{4/3} d}-\frac{c^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d (b c-a d)^{4/3}}-$$

$$\frac{c^{4/3} \text{Log}\left[c+d x^3\right]}{6 d (b c-a d)^{4/3}}+\frac{c^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-(a+b x^3)^{1/3}\right]}{2 d (b c-a d)^{4/3}}-\frac{\text{Log}\left[-b^{1/3} x+(a+b x^3)^{1/3}\right]}{2 b^{4/3} d}$$

Result (type 6, 67 leaves, 2 steps) :

$$\frac{x^7 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{7 a c (a+b x^3)^{1/3}}$$

Problem 756: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{(a+b x^3)^{4/3} (c+d x^3)} \, dx$$

Optimal (type 3, 172 leaves, 3 steps) :

$$-\frac{x}{(b c-a d) \left(a+b x^3\right)^{1/3}}+\frac{c^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c-a d)^{4/3}}+\frac{c^{1/3} \text{Log}\left[c+d x^3\right]}{6 (b c-a d)^{4/3}}-\frac{c^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 (b c-a d)^{4/3}}$$

Result (type 5, 92 leaves, 2 steps):

$$\frac{x^4 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{4}{3},\frac{4}{3},\frac{7}{3},-\frac{c \left(\frac{b x^3}{a}-\frac{d x^3}{c}\right)}{c+d x^3}\right]}{4 a c \left(a+b x^3\right)^{1/3} \left(1+\frac{d x^3}{c}\right)^{4/3}}$$

Problem 757: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 179 leaves, 2 steps):

$$\frac{b x}{a (b c-a d) \left(a+b x^3\right)^{1/3}}-\frac{d \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} (b c-a d)^{4/3}}-\frac{d \text{Log}\left[c+d x^3\right]}{6 c^{2/3} (b c-a d)^{4/3}}+\frac{d \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} (b c-a d)^{4/3}}$$

Result (type 3, 238 leaves, 8 steps):

$$\frac{b x}{a (b c-a d) \left(a+b x^3\right)^{1/3}}-\frac{d \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{2/3} (b c-a d)^{4/3}}+\frac{d \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{2/3} (b c-a d)^{4/3}}-\frac{d \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{2/3} (b c-a d)^{4/3}}$$

Problem 758: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 229 leaves, 4 steps):

$$\frac{b}{a (b c-a d) x^2 \left(a+b x^3\right)^{1/3}}-\frac{(3 b c-a d) \left(a+b x^3\right)^{2/3}}{2 a^2 c (b c-a d) x^2}+\frac{d^2 \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{5/3} (b c-a d)^{4/3}}+\frac{d^2 \text{Log}\left[c+d x^3\right]}{6 c^{5/3} (b c-a d)^{4/3}}-\frac{d^2 \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{5/3} (b c-a d)^{4/3}}$$

Result (type 5, 542 leaves, 2 steps):

$$\frac{1}{14\,c^4\,(b\,c-a\,d)\,x^5\,(a+b\,x^3)^{7/3}}\Bigg(28\,c^4\,(a+b\,x^3)^2+168\,c^3\,d\,x^3\,(a+b\,x^3)^2+126\,c^2\,d^2\,x^6\,(a+b\,x^3)^2-28\,c^4\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$

$$168\,c^3\,d\,x^3\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-126\,c^2\,d^2\,x^6\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$

$$15\,c^2\,(b\,c-a\,d)^2\,x^6\,\text{Hypergeometric2F1}\Big[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-42\,c\,d\,(b\,c-a\,d)^2\,x^9\,\text{Hypergeometric2F1}\Big[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$

$$27\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{Hypergeometric2F1}\Big[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-9\,c^2\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{7}{3}\Big\},\Big\{1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$

$$18\,c\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{7}{3}\Big\},\Big\{1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$

$$9\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{7}{3}\Big\},\Big\{1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]\Bigg)$$

Problem 759: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6\,(a+b\,x^3)^{4/3}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 287 leaves, 5 steps):

$$\frac{b}{a\,(b\,c-a\,d)\,x^5\,(a+b\,x^3)^{1/3}}-\frac{(6\,b\,c-a\,d)\,(a+b\,x^3)^{2/3}}{5\,a^2\,c\,(b\,c-a\,d)\,x^5}+\frac{(18\,b^2\,c^2-3\,a\,b\,c\,d-5\,a^2\,d^2)\,(a+b\,x^3)^{2/3}}{10\,a^3\,c^2\,(b\,c-a\,d)\,x^2}-$$

$$\frac{d^3\,\text{ArcTan}\Big[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\Big]}{\sqrt{3}\,c^{8/3}\,(b\,c-a\,d)^{4/3}}-\frac{d^3\,\text{Log}\big[c+d\,x^3\big]}{6\,c^{8/3}\,(b\,c-a\,d)^{4/3}}+\frac{d^3\,\text{Log}\Big[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\Big]}{2\,c^{8/3}\,(b\,c-a\,d)^{4/3}}$$

Result (type 5, 950 leaves, 2 steps):

$$\frac{1}{70\,c^5\,(b\,c-a\,d)\,x^8\,(a+b\,x^3)^{7/3}}\Bigg(56\,c^5\,(a+b\,x^3)^2-252\,c^4\,d\,x^3\,(a+b\,x^3)^2-1512\,c^3\,d^2\,x^6\,(a+b\,x^3)^2-1134\,c^2\,d^3\,x^9\,(a+b\,x^3)^2-$$
$$56\,c^5\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+252\,c^4\,d\,x^3\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$1512\,c^3\,d^2\,x^6\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+1134\,c^2\,d^3\,x^9\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$18\,c^3\,(b\,c-a\,d)^2\,x^6\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+171\,c^2\,d\,(b\,c-a\,d)^2\,x^9\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$486\,c\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+297\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$27\,c^3\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$216\,c^2\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$351\,c\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$162\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$27\,c^3\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$81\,c^2\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$81\,c\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$27\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\Bigg)$$

Problem 760: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x^9\,(a+b\,x^3)^{4/3}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 351 leaves, 6 steps):

$$\frac{b}{a\,(b\,c-a\,d)\,x^8\,(a+b\,x^3)^{1/3}}-\frac{(9\,b\,c-a\,d)\,(a+b\,x^3)^{2/3}}{8\,a^2\,c\,(b\,c-a\,d)\,x^8}+\frac{(9\,b\,c-4\,a\,d)\,(3\,b\,c+a\,d)\,(a+b\,x^3)^{2/3}}{20\,a^3\,c^2\,(b\,c-a\,d)\,x^5}-$$

$$\frac{(81\,b^3\,c^3-9\,a\,b^2\,c^2\,d-12\,a^2\,b\,c\,d^2-20\,a^3\,d^3)\,(a+b\,x^3)^{2/3}}{40\,a^4\,c^3\,(b\,c-a\,d)\,x^2}+\frac{d^4\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{11/3}\,(b\,c-a\,d)^{4/3}}+\frac{d^4\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{11/3}\,(b\,c-a\,d)^{4/3}}-\frac{d^4\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\right]}{2\,c^{11/3}\,(b\,c-a\,d)^{4/3}}$$

Result (type 5, 1486 leaves, 2 steps):

$$\frac{1}{560\,c^6\,(b\,c-a\,d)\,x^{11}\,(a+b\,x^3)^{7/3}}$$

$$\Big(280\,c^6\,(a+b\,x^3)^2-672\,c^5\,d\,x^3\,(a+b\,x^3)^2+3024\,c^4\,d^2\,x^6\,(a+b\,x^3)^2+18\,144\,c^3\,d^3\,x^9\,(a+b\,x^3)^2+13\,608\,c^2\,d^4\,x^{12}\,(a+b\,x^3)^2-$$

$$280\,c^6\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+672\,c^5\,d\,x^3\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$

$$3024\,c^4\,d^2\,x^6\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-18\,144\,c^3\,d^3\,x^9\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$

$$13\,608\,c^2\,d^4\,x^{12}\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-66\,c^4\,(b\,c-a\,d)^2\,x^6\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$

$$312\,c^3\,d\,(b\,c-a\,d)^2\,x^9\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-2268\,c^2\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$

$$6696\,c\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-4050\,d^4\,(b\,c-a\,d)^2\,x^{18}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$

$$189\,c^4\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$

$$108\,c^3\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$

$$3618\,c^2\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$

$$6156\,c\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$

$$2835\,d^4\,(b\,c-a\,d)^2\,x^{18}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$

$54 c^4 (b c - a d)^2 x^6 \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$

$648 c^3 d (b c - a d)^2 x^9 \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$

$2268 c^2 d^2 (b c - a d)^2 x^{12} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$

$2376 c d^3 (b c - a d)^2 x^{15} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$

$810 d^4 (b c - a d)^2 x^{18} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$

$81 c^4 (b c - a d)^2 x^6 \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$

$324 c^3 d (b c - a d)^2 x^9 \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$

$486 c^2 d^2 (b c - a d)^2 x^{12} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$

$324 c d^3 (b c - a d)^2 x^{15} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$

$81 d^4 (b c - a d)^2 x^{18} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]\Big)$

Test results for the 46 problems in "1.1.3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r.m"

Test results for the 594 problems in "1.1.3.8 P(x) (c x)^m (a+b x^n)^p.m"

Test results for the 454 problems in "1.1.4.2 (c x)^m (a x^j+b x^n)^p.m"

Test results for the 298 problems in "1.1.4.3 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q.m"

Test results for the 143 problems in "1.2.1.1 (a+b x+c x^2)^p.m"

Test results for the 2590 problems in "1.2.1.2 (d+e x)^m (a+b x+c x^2)^p.m"

Problem 1412: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x+c x^2\right)^{4/3}}{(b d+2 c d x)^{11/3}} \, dx$$

Optimal (type 3, 320 leaves, 8 steps):

$$-\frac{3 (d (b+2 c x))^{4/3} \left(a+b x+c x^2\right)^{1/3}}{16 c^2 \left(b^2-4 a c\right) d^5}+\frac{9 (d (b+2 c x))^{4/3} \left(a+b x+c x^2\right)^{4/3}}{16 c \left(b^2-4 a c\right)^2 d^5}+\frac{3 \left(a+b x+c x^2\right)^{7/3}}{4 \left(b^2-4 a c\right) d (b d+2 c d x)^{8/3}}-\frac{9 \left(a+b x+c x^2\right)^{7/3}}{4 \left(b^2-4 a c\right)^2 d^3 (b d+2 c d x)^{2/3}}-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1+\frac{2^{1/3} (d (b+2 c x))^{2/3}}{c^{1/3} d^{2/3} \left(a+b x+c x^2\right)^{1/3}}}{\sqrt{3}}\right]}{16\times 2^{2/3} c^{7/3} d^{11/3}}-\frac{3 \operatorname{Log}\left[(d (b+2 c x))^{2/3}-2^{2/3} c^{1/3} d^{2/3} \left(a+b x+c x^2\right)^{1/3}\right]}{32\times 2^{2/3} c^{7/3} d^{11/3}}$$

Result (type 3, 468 leaves, 14 steps):

$$-\frac{3 (d (b+2 c x))^{4/3} \left(a+b x+c x^2\right)^{1/3}}{16 c^2 \left(b^2-4 a c\right) d^5}+\frac{9 (d (b+2 c x))^{4/3} \left(a+b x+c x^2\right)^{4/3}}{16 c \left(b^2-4 a c\right)^2 d^5}+$$

$$\frac{3 \left(a+b x+c x^2\right)^{7/3}}{4 \left(b^2-4 a c\right) d (b d+2 c d x)^{8/3}}-\frac{9 \left(a+b x+c x^2\right)^{7/3}}{4 \left(b^2-4 a c\right)^2 d^3 (b d+2 c d x)^{2/3}}-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{c^{1/3} d^{2/3}+\frac{2^{1/3} (d (b+2 c x))^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}}{\sqrt{3} c^{1/3} d^{2/3}}\right]}{16\times 2^{2/3} c^{7/3} d^{11/3}}-$$

$$\frac{\operatorname{Log}\left[-\frac{2^{1/3} (d (b+2 c x))^{2/3}-2 c^{1/3} d^{2/3} \left(a+b x+c x^2\right)^{1/3}}{\left(a+b x+c x^2\right)^{1/3}}\right]}{16\times 2^{2/3} c^{7/3} d^{11/3}}+\frac{\operatorname{Log}\left[\frac{(d (b+2 c x))^{4/3}+2^{2/3} c^{1/3} d^{2/3} (d (b+2 c x))^{2/3} \left(a+b x+c x^2\right)^{1/3}+2\times 2^{1/3} c^{2/3} d^{4/3} \left(a+b x+c x^2\right)^{2/3}}{\left(a+b x+c x^2\right)^{2/3}}\right]}{32\times 2^{2/3} c^{7/3} d^{11/3}}$$

Test results for the 2646 problems in "1.2.1.3 (d+e x)^m (f+g x) (a+b x+c x^2)^p.m"

Test results for the 958 problems in "1.2.1.4 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p.m"

Problem 833: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{-1+x} \sqrt{1+x}}{1+x-x^2} \, dx$$

Optimal (type 3, 91 leaves, ? steps):

$$-\text{ArcCosh}[x]+\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\ \sqrt{-1+x}}\right]+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}\ \sqrt{-1+x}}\right]$$

Result (type 3, 191 leaves, 9 steps):

$$\frac{\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \sqrt{-1+x}\ \sqrt{1+x}\ \text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}-$$

$$\frac{\sqrt{-1+x}\ \sqrt{1+x}\ \text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}-\frac{\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \sqrt{-1+x}\ \sqrt{1+x}\ \text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}$$

Test results for the 123 problems in "1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Test results for the 143 problems in "1.2.1.6 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Test results for the 400 problems in "1.2.1.9 P(x) (d+e x)^m (a+b x+c x^2)^p.m"

Test results for the 1126 problems in "1.2.2.2 (d x)^m (a+b x^2+c x^4)^p.m"

Test results for the 413 problems in "1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p.m"

Problem 174: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\,x^2}}{\sqrt{1-x^4}}\,dx$$

Optimal (type 4, 112 leaves, ? steps):

$$\frac{a\sqrt{1-x^2}\,\sqrt{\frac{a\,(1+x^2)}{a+b\,x^2}}\;\text{EllipticPi}\left[\frac{b}{a+b},\text{ArcSin}\left[\frac{\sqrt{a+b}\;x}{\sqrt{a+b\,x^2}}\right],-\frac{a-b}{a+b}\right]}{\sqrt{a+b}\;\sqrt{1+x^2}\;\sqrt{\frac{a\,(1-x^2)}{a+b\,x^2}}}$$

Result (type 8, 25 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{\sqrt{a+b\,x^2}}{\sqrt{1-x^4}},x\right]$$

Test results for the 413 problems in "1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Problem 374: Result valid but suboptimal antiderivative.

$$\int\frac{\left(d+e\,x^2\right)^{3/2}}{x^2\,\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 260 leaves, ? steps):

$$-\frac{d\sqrt{d+e\,x^2}}{a\,x}-\frac{\left(2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e\right)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e}\;x}{\sqrt{b-\sqrt{b^2-4\,a\,c}}\;\sqrt{d+e\,x^2}}\right]}{\sqrt{b^2-4\,a\,c}\;\left(b-\sqrt{b^2-4\,a\,c}\right)^{3/2}}+\frac{\left(2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e\right)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\;x}{\sqrt{b+\sqrt{b^2-4\,a\,c}}\;\sqrt{d+e\,x^2}}\right]}{\sqrt{b^2-4\,a\,c}\;\left(b+\sqrt{b^2-4\,a\,c}\right)^{3/2}}$$

Result (type 3, 432 leaves, 16 steps) :

$$-\frac{d\sqrt{d+e x^2}}{a x}-\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{2 a \sqrt{b-\sqrt{b^2-4 a c}}}-$$

$$\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{2 a \sqrt{b+\sqrt{b^2-4 a c}}}+\frac{d\sqrt{e}\ \text{ArcTanh}\left[\frac{\sqrt{e}\ x}{\sqrt{d+e x^2}}\right]}{a}-$$

$$\frac{\sqrt{e}\left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e}\ x}{\sqrt{d+e x^2}}\right]}{2 a}-\frac{\sqrt{e}\left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e}\ x}{\sqrt{d+e x^2}}\right]}{2 a}$$

Test results for the 111 problems in "1.2.2.5 P(x) (a+b x^2+c x^4)^p.m"

Test results for the 145 problems in "1.2.2.6 P(x) (d x)^m (a+b x^2+c x^4)^p.m"

Test results for the 42 problems in "1.2.2.7 P(x) (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Test results for the 4 problems in "1.2.2.8 P(x) (d+e x)^q (a+b x^2+c x^4)^p.m"

Test results for the 664 problems in "1.2.3.2 (d x)^m (a+b x^n+c x^(2 n))^p.m"

Problem 24: Result valid but suboptimal antiderivative.

$$\int x^8 \left(a^2+2 a b x^3+b^2 x^6\right)^{3/2} dx$$

Optimal (type 2, 119 leaves, ? steps):

$$\frac{a^2 \left(a+b x^3\right)^3 \sqrt{a^2+2 a b x^3+b^2 x^6}}{12 b^3}-\frac{2 a \left(a+b x^3\right)^4 \sqrt{a^2+2 a b x^3+b^2 x^6}}{15 b^3}+\frac{\left(a+b x^3\right)^5 \sqrt{a^2+2 a b x^3+b^2 x^6}}{18 b^3}$$

Result (type 2, 167 leaves, 4 steps):

$$\frac{a^3 x^9 \sqrt{a^2+2 a b x^3+b^2 x^6}}{9 \left(a+b x^3\right)}+\frac{a^2 b x^{12} \sqrt{a^2+2 a b x^3+b^2 x^6}}{4 \left(a+b x^3\right)}+\frac{a b^2 x^{15} \sqrt{a^2+2 a b x^3+b^2 x^6}}{5 \left(a+b x^3\right)}+\frac{b^3 x^{18} \sqrt{a^2+2 a b x^3+b^2 x^6}}{18 \left(a+b x^3\right)}$$

Problem 478: Result unnecessarily involves higher level functions.

$$\int \left(\frac{\left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{x^2}-\frac{2 b^3 (1-2 p) (1-p) p \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{3 a^3 x}\right) dx$$

Optimal (type 3, 146 leaves, ? steps):

$$-\frac{\left(a+b x^{1/3}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{a x}+\frac{b (1-p) \left(a+b x^{1/3}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{a^2 x^{2/3}}-\frac{b^2 (1-2 p) (1-p) \left(a+b x^{1/3}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{a^3 x^{1/3}}$$

Result (type 5, 162 leaves, 7 steps):

$$\frac{1}{a^3 (1+2 p)} 2 b^3 (1-2 p) (1-p) p \left(1+\frac{b x^{1/3}}{a}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p \text{Hypergeometric2F1}\left[1, 1+2 p, 2 (1+p), 1+\frac{b x^{1/3}}{a}\right]+$$
$$\frac{3 b^3 \left(1+\frac{b x^{1/3}}{a}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p \text{Hypergeometric2F1}\left[4, 1+2 p, 2 (1+p), 1+\frac{b x^{1/3}}{a}\right]}{a^3 (1+2 p)}$$

Test results for the 96 problems in "1.2.3.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 156 problems in "1.2.3.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 17 problems in "1.2.3.5 P(x) (d x)^m (a+b x^n+c x^(2 n))^p.m"

Test results for the 140 problems in "1.2.4.2 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p.m"

Test results for the 494 problems in "1.3.1 Rational functions.m"

Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(b x^{1+p} \left(b x+c x^3\right)^p+2 c x^{3+p} \left(b x+c x^3\right)^p\right) dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\frac{x^{1+p}\,\left(b\,x+c\,x^3\right)^{1+p}}{2\,\left(1+p\right)}$$

Result (type 5, 116 leaves, 7 steps):

$$\frac{b\,x^{2+p}\,\left(1+\frac{c\,x^2}{b}\right)^{-p}\,\left(b\,x+c\,x^3\right)^p\,\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,-\frac{c\,x^2}{b}\right]}{2\,\left(1+p\right)}+\frac{c\,x^{4+p}\,\left(1+\frac{c\,x^2}{b}\right)^{-p}\,\left(b\,x+c\,x^3\right)^p\,\text{Hypergeometric2F1}\left[-p,\,2+p,\,3+p,\,-\frac{c\,x^2}{b}\right]}{2+p}$$

Problem 221: Result valid but suboptimal antiderivative.

$$\int\left(1+2\,x\right)\,\left(x+x^2\right)^3\,\left(-18+7\,\left(x+x^2\right)^3\right)^2\,dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81\,x^4\,\left(1+x\right)^4-36\,x^7\,\left(1+x\right)^7+\frac{49}{10}\,x^{10}\,\left(1+x\right)^{10}$$

Result (type 1, 96 leaves, 3 steps):

$$81\,x^4+324\,x^5+486\,x^6+288\,x^7-171\,x^8-756\,x^9-\frac{12\,551\,x^{10}}{10}-1211\,x^{11}-\frac{1071\,x^{12}}{2}+336\,x^{13}+993\,x^{14}+\frac{6174\,x^{15}}{5}+1029\,x^{16}+588\,x^{17}+\frac{441\,x^{18}}{2}+49\,x^{19}+\frac{49\,x^{20}}{10}$$

Problem 222: Result valid but suboptimal antiderivative.

$$\int x^3\,\left(1+x\right)^3\,\left(1+2\,x\right)\,\left(-18+7\,x^3\,\left(1+x\right)^3\right)^2\,dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81\,x^4\,\left(1+x\right)^4-36\,x^7\,\left(1+x\right)^7+\frac{49}{10}\,x^{10}\,\left(1+x\right)^{10}$$

Result (type 1, 96 leaves, 2 steps):

$$81\,x^4+324\,x^5+486\,x^6+288\,x^7-171\,x^8-756\,x^9-\frac{12\,551\,x^{10}}{10}-1211\,x^{11}-\frac{1071\,x^{12}}{2}+336\,x^{13}+993\,x^{14}+\frac{6174\,x^{15}}{5}+1029\,x^{16}+588\,x^{17}+\frac{441\,x^{18}}{2}+49\,x^{19}+\frac{49\,x^{20}}{10}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{-20 x + 4 x^2}{9 - 10 x^2 + x^4} \, dx$$

Optimal (type 3, 31 leaves, ? steps):

$$\text{Log}[1 - x] - \frac{1}{2} \text{Log}[3 - x] + \frac{3}{2} \text{Log}[1 + x] - 2 \text{Log}[3 + x]$$

Result (type 3, 41 leaves, 11 steps):

$$-\frac{3}{2} \text{ArcTanh}\left[\frac{x}{3}\right] + \frac{\text{ArcTanh}[x]}{2} + \frac{5}{4} \text{Log}\left[1 - x^2\right] - \frac{5}{4} \text{Log}\left[9 - x^2\right]$$

Problem 393: Unable to integrate problem.

$$\int \frac{\left(1 + x^2\right)^2}{a x^6 + b \left(1 + x^2\right)^3} \, dx$$

Optimal (type 3, 168 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+b^{1/3}}\, x}{b^{1/6}}\right]}{3 \sqrt{a^{1/3} + b^{1/3}}\, b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{-(-1)^{1/3} a^{1/3}+b^{1/3}}\, x}{b^{1/6}}\right]}{3 \sqrt{-(-1)^{1/3} a^{1/3} + b^{1/3}}\, b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-1)^{2/3} a^{1/3}+b^{1/3}}\, x}{b^{1/6}}\right]}{3 \sqrt{(-1)^{2/3} a^{1/3} + b^{1/3}}\, b^{5/6}}$$

Result (type 8, 68 leaves, 5 steps):

$$\text{CannotIntegrate}\left[\frac{1}{a x^6 + b \left(1 + x^2\right)^3}, x\right] + 2\, \text{CannotIntegrate}\left[\frac{x^2}{a x^6 + b \left(1 + x^2\right)^3}, x\right] + \text{CannotIntegrate}\left[\frac{x^4}{a x^6 + b \left(1 + x^2\right)^3}, x\right]$$

Problem 493: Unable to integrate problem.

$$\int \left(\frac{3 \left(-47 + 228 x + 120 x^2 + 19 x^3\right)}{\left(3 + x + x^4\right)^4} + \frac{42 - 320 x - 75 x^2 - 8 x^3}{\left(3 + x + x^4\right)^3} + \frac{30 x}{\left(3 + x + x^4\right)^2}\right) dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2 - 3 x + 5 x^2 + x^4 - 5 x^6}{\left(3 + x + x^4\right)^3}$$

Result (type 8, 121 leaves, 7 steps):

$$-\frac{19}{4\left(3+x+x^4\right)^3}+\frac{1}{\left(3+x+x^4\right)^2}-\frac{621}{4}\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^4},x\right]+684\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^4},x\right]+360\,\text{CannotIntegrate}\left[\frac{x^2}{\left(3+x+x^4\right)^4},x\right]+44\,\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^3},x\right]-320\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^3},x\right]-75\,\text{CannotIntegrate}\left[\frac{x^2}{\left(3+x+x^4\right)^3},x\right]+30\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^2},x\right]$$

Problem 494: Unable to integrate problem.

$$\int\left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right)dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2-3x+5x^2+x^4-5x^6}{\left(3+x+x^4\right)^3}$$

Result (type 8, 177 leaves, 13 steps):

$$\frac{7}{2\left(3+x+x^4\right)^3}-\frac{63x}{22\left(3+x+x^4\right)^3}-\frac{12x^2}{\left(3+x+x^4\right)^3}-\frac{5x^3}{\left(3+x+x^4\right)^3}+\frac{3x^4}{2\left(3+x+x^4\right)^3}-\frac{10x^6}{\left(3+x+x^4\right)^3}-\frac{1}{2\left(3+x+x^4\right)^2}+\frac{5x^2}{\left(3+x+x^4\right)^2}+\frac{144}{11}\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^4},x\right]+\frac{828}{11}\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^4},x\right]+18\,\text{CannotIntegrate}\left[\frac{x^2}{\left(3+x+x^4\right)^4},x\right]-4\,\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^3},x\right]-20\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^3},x\right]$$

Test results for the 1025 problems in "1.3.2 Algebraic functions.m"

Problem 20: Unable to integrate problem.

$$\int\frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 187 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 d x}{\left(2 c^3+d^3 x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2 \sqrt{3}\, c^2 d}+\frac{\sqrt{3}\, \text{ArcTan}\left[\frac{1+\frac{2 (2 c+d x)}{\left(2 c^3+d^3 x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2 c^2 d}-\frac{\text{Log}[c+d x]}{2 c^2 d}-\frac{\text{Log}\left[d x-\left(2 c^3+d^3 x^3\right)^{1/3}\right]}{4 c^2 d}+\frac{3 \text{Log}\left[d (2 c+d x)-d \left(2 c^3+d^3 x^3\right)^{1/3}\right]}{4 c^2 d}$$

Result (type 8, 27 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x) \left(2 c^3+d^3 x^3\right)^{2/3}}, x\right]$$

Problem 21: Unable to integrate problem.

$$\int \frac{1}{\left(1+2^{1/3} x\right) \left(1+x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 147 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 x}{\left(1+x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\sqrt{3}\, \text{ArcTan}\left[\frac{1+\frac{2 \left(2^{2/3}+x\right)}{\left(1+x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}-\frac{\text{Log}\left[1+2^{1/3} x\right]}{2^{2/3}}-\frac{\text{Log}\left[x-\left(1+x^3\right)^{1/3}\right]}{2\times 2^{2/3}}+\frac{3 \text{Log}\left[2+2^{1/3} x-2^{1/3} \left(1+x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{\left(1+2^{1/3} x\right) \left(1+x^3\right)^{2/3}}, x\right]$$

Problem 22: Unable to integrate problem.

$$\int \frac{1}{\left(1-2^{1/3} x\right) \left(1-x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 159 leaves, 1 step):

$$-\frac{\sqrt{3}\, \text{ArcTan}\left[\frac{1+\frac{2\times 2^{2/3}-2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\text{Log}\left[1-2^{1/3} x\right]}{2^{2/3}}+\frac{\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}-\frac{3 \text{Log}\left[-2+2^{1/3} x+2^{1/3} \left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 26 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{\left(1-2^{1/3} x\right) \left(1-x^3\right)^{2/3}}, x\right]$$

Problem 23: Result valid but suboptimal antiderivative.

$$\int (c+d x)^4 \left(a+b x^3\right)^{1/3} \, dx$$

Optimal (type 5, 387 leaves, 11 steps):

$$\frac{3 a c^2 d^2 \left(a+b x^3\right)^{1/3}}{2 b}+\frac{a d^4 x^2 \left(a+b x^3\right)^{1/3}}{18 b}+\frac{1}{30} \left(a+b x^3\right)^{1/3} \left(15 c^4 x+40 c^3 d x^2+45 c^2 d^2 x^3+24 c d^3 x^4+5 d^4 x^5\right)-\frac{4 a c^3 d \,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{2/3}}+\frac{a^2 d^4 \,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{5/3}}+\frac{a c^4 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 \left(a+b x^3\right)^{2/3}}+\frac{a c d^3 x^4 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{b x^3}{a}\right]}{5 \left(a+b x^3\right)^{2/3}}-\frac{2 a c^3 d \,\text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{3 b^{2/3}}+\frac{a^2 d^4 \,\text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{18 b^{5/3}}$$

Result (type 5, 498 leaves, 23 steps):

$$\frac{3 a c^2 d^2 \left(a+b x^3\right)^{1/3}}{2 b}+\frac{a d^4 x^2 \left(a+b x^3\right)^{1/3}}{18 b}+\frac{1}{30} \left(a+b x^3\right)^{1/3} \left(15 c^4 x+40 c^3 d x^2+45 c^2 d^2 x^3+24 c d^3 x^4+5 d^4 x^5\right)-\frac{4 a c^3 d \,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{2/3}}+\frac{a^2 d^4 \,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{5/3}}+\frac{a c^4 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 \left(a+b x^3\right)^{2/3}}+\frac{a c d^3 x^4 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{b x^3}{a}\right]}{5 \left(a+b x^3\right)^{2/3}}-\frac{4 a c^3 d \,\text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{2/3}}+\frac{a^2 d^4 \,\text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{27 b^{5/3}}+\frac{2 a c^3 d \,\text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{2/3}}-\frac{a^2 d^4 \,\text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{54 b^{5/3}}$$

Problem 24: Result valid but suboptimal antiderivative.

$$\int (c+d x)^3 \left(a+b x^3\right)^{1/3} \, dx$$

Optimal (type 5, 242 leaves, 9 steps):

$$\frac{3 a c d^2 \left(a+b x^3\right)^{1/3}}{4 b}+\frac{a d^3 x \left(a+b x^3\right)^{1/3}}{10 b}+\frac{1}{20} \left(a+b x^3\right)^{1/3} \left(10 c^3 x+20 c^2 d x^2+15 c d^2 x^3+4 d^3 x^4\right)-\frac{a c^2 d \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3}}+\frac{a \left(5 b c^3-a d^3\right) x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{10 b \left(a+b x^3\right)^{2/3}}-\frac{a c^2 d \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 b^{2/3}}$$

Result (type 5, 297 leaves, 15 steps):

$$\frac{3 a c d^2 \left(a+b x^3\right)^{1/3}}{4 b}+\frac{a d^3 x \left(a+b x^3\right)^{1/3}}{10 b}+\frac{1}{20} \left(a+b x^3\right)^{1/3} \left(10 c^3 x+20 c^2 d x^2+15 c d^2 x^3+4 d^3 x^4\right)-\frac{a c^2 d \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3}}+\frac{a \left(5 b c^3-a d^3\right) x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{10 b \left(a+b x^3\right)^{2/3}}-\frac{a c^2 d \text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{2/3}}+\frac{a c^2 d \text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 b^{2/3}}$$

Problem 25: Result valid but suboptimal antiderivative.

$$\int (c+d x)^2 \left(a+b x^3\right)^{1/3} \, dx$$

Optimal (type 5, 192 leaves, 8 steps):

$$\frac{a d^2 \left(a+b x^3\right)^{1/3}}{4 b}+\frac{1}{12} \left(a+b x^3\right)^{1/3} \left(6 c^2 x+8 c d x^2+3 d^2 x^3\right)-\frac{2 a c d \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{2/3}}+\frac{a c^2 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 \left(a+b x^3\right)^{2/3}}-\frac{a c d \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{3 b^{2/3}}$$

Result (type 5, 245 leaves, 14 steps):

$$\frac{a d^2 \left(a+b x^3\right)^{1/3}}{4 b}+\frac{1}{12} \left(a+b x^3\right)^{1/3} \left(6 c^2 x+8 c d x^2+3 d^2 x^3\right)-\frac{2 a c d \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{2/3}}+\frac{a c^2 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 \left(a+b x^3\right)^{2/3}}-\frac{2 a c d \text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{2/3}}+\frac{a c d \text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{2/3}}$$

Problem 26: Result valid but suboptimal antiderivative.

$$\int (c+d x) \left(a+b x^3\right)^{1/3} dx$$

Optimal (type 5, 155 leaves, 6 steps):

$$\frac{1}{6}\left(3 c x+2 d x^2\right)\left(a+b x^3\right)^{1/3}-\frac{a d \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}\, b^{2/3}}+\frac{a c x\left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2\left(a+b x^3\right)^{2/3}}-\frac{a d \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 b^{2/3}}$$

Result (type 5, 207 leaves, 12 steps):

$$\frac{1}{6}\left(3 c x+2 d x^2\right)\left(a+b x^3\right)^{1/3}-\frac{a d \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}\, b^{2/3}}+$$

$$\frac{a c x\left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2\left(a+b x^3\right)^{2/3}}-\frac{a d \operatorname{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{2/3}}+\frac{a d \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{18 b^{2/3}}$$

Problem 27: Unable to integrate problem.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{c+d x} dx$$

Optimal (type 6, 435 leaves, 13 steps):

$$\frac{\left(a+b x^3\right)^{1/3}}{d}+\frac{x\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c\left(1+\frac{b x^3}{a}\right)^{1/3}}+\frac{b^{1/3} c \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^2}-$$

$$\frac{\left(b c^3-a d^3\right)^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2\left(b c^3-a d^3\right)^{1/3} x}{c\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^2}+\frac{\left(b c^3-a d^3\right)^{1/3} \operatorname{ArcTan}\left[\frac{1-\frac{2 d\left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^2}+\frac{\left(b c^3-a d^3\right)^{1/3} \operatorname{Log}\left[c^3+d^3 x^3\right]}{3 d^2}+$$

$$\frac{b^{1/3} c \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 d^2}-\frac{\left(b c^3-a d^3\right)^{1/3} \operatorname{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{2 d^2}-\frac{\left(b c^3-a d^3\right)^{1/3} \operatorname{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d\left(a+b x^3\right)^{1/3}\right]}{2 d^2}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{(a+b\,x^3)^{1/3}}{c+d\,x},\,x\right]$$

Problem 28: Unable to integrate problem.

$$\int \frac{(a+b\,x^3)^{1/3}}{(c+d\,x)^2}\,dx$$

Optimal (type 6, 818 leaves, 20 steps):

$$-\frac{c^2\,(a+b\,x^3)^{1/3}}{d\,(c^3+d^3\,x^3)}-\frac{d\,x^2\,(a+b\,x^3)^{1/3}}{c^3+d^3\,x^3}+\frac{x\,(a+b\,x^3)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d^3\,x^3}{c^3}\right]}{c^2\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}-\frac{d^3\,x^4\,(a+b\,x^3)^{1/3}\,\text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d^3\,x^3}{c^3}\right]}{2\,c^5\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}-$$

$$\frac{b^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2}+\frac{2\,a\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c^3-a\,d^3)^{1/3}\,x}{c\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,c\,(b\,c^3-a\,d^3)^{2/3}}+\frac{(3\,b\,c^3-2\,a\,d^3)\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c^3-a\,d^3)^{1/3}\,x}{c\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,c\,d^2\,(b\,c^3-a\,d^3)^{2/3}}-\frac{b\,c^2\,\text{ArcTan}\left[\frac{1-\frac{2\,d\,(a+b\,x^3)^{1/3}}{(b\,c^3-a\,d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2\,(b\,c^3-a\,d^3)^{2/3}}-$$

$$\frac{b\,c^2\,\text{Log}\left[c^3+d^3\,x^3\right]}{6\,d^2\,(b\,c^3-a\,d^3)^{2/3}}-\frac{a\,d\,\text{Log}\left[c^3+d^3\,x^3\right]}{9\,c\,(b\,c^3-a\,d^3)^{2/3}}-\frac{(3\,b\,c^3-2\,a\,d^3)\,\text{Log}\left[c^3+d^3\,x^3\right]}{18\,c\,d^2\,(b\,c^3-a\,d^3)^{2/3}}-\frac{b^{1/3}\,\text{Log}\left[b^{1/3}\,x-(a+b\,x^3)^{1/3}\right]}{2\,d^2}+$$

$$\frac{a\,d\,\text{Log}\left[\frac{(b\,c^3-a\,d^3)^{1/3}\,x}{c}-(a+b\,x^3)^{1/3}\right]}{3\,c\,(b\,c^3-a\,d^3)^{2/3}}+\frac{(3\,b\,c^3-2\,a\,d^3)\,\text{Log}\left[\frac{(b\,c^3-a\,d^3)^{1/3}\,x}{c}-(a+b\,x^3)^{1/3}\right]}{6\,c\,d^2\,(b\,c^3-a\,d^3)^{2/3}}+\frac{b\,c^2\,\text{Log}\left[(b\,c^3-a\,d^3)^{1/3}+d\,(a+b\,x^3)^{1/3}\right]}{2\,d^2\,(b\,c^3-a\,d^3)^{2/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{(a+b\,x^3)^{1/3}}{(c+d\,x)^2},\,x\right]$$

Problem 33: Unable to integrate problem.

$$\int \frac{1}{(c+d\,x)\,(a+b\,x^3)^{1/3}}\,dx$$

Optimal (type 6, 333 leaves, 10 steps):

$$-\frac{d\,x^2\left(1+\frac{b x^3}{a}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{2\,c^2\left(a+b x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2\left(b c^3-a d^3\right)^{1/3}x}{c\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\left(b c^3-a d^3\right)^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2 d\left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\left(b c^3-a d^3\right)^{1/3}}+\frac{\text{Log}\left[c^3+d^3 x^3\right]}{3\left(b c^3-a d^3\right)^{1/3}}-\frac{\text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3}x}{c}-\left(a+b x^3\right)^{1/3}\right]}{2\left(b c^3-a d^3\right)^{1/3}}-\frac{\text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d\left(a+b x^3\right)^{1/3}\right]}{2\left(b c^3-a d^3\right)^{1/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c+d x)\left(a+b x^3\right)^{1/3}},x\right]$$

Problem 34: Unable to integrate problem.

$$\int\frac{1}{(c+d x)^2\left(a+b x^3\right)^{1/3}}\,dx$$

Optimal (type 6, 761 leaves, 17 steps):

$$\frac{c^2 d^2\left(a+b x^3\right)^{2/3}}{\left(b c^3-a d^3\right)\left(c^3+d^3 x^3\right)}-\frac{c\,d^3 x\left(a+b x^3\right)^{2/3}}{\left(b c^3-a d^3\right)\left(c^3+d^3 x^3\right)}-\frac{d\,x^2\left(1+\frac{b x^3}{a}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},2,\frac{5}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c^3\left(a+b x^3\right)^{1/3}}+\frac{d^4 x^5\left(1+\frac{b x^3}{a}\right)^{1/3}\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{5\,c^6\left(a+b x^3\right)^{1/3}}+\frac{2\,a\,d^3\,\text{ArcTan}\left[\frac{1+\frac{2\left(b c^3-a d^3\right)^{1/3}x}{c\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}\,c\left(b c^3-a d^3\right)^{4/3}}+\frac{\left(3 b c^3-2 a d^3\right)\text{ArcTan}\left[\frac{1+\frac{2\left(b c^3-a d^3\right)^{1/3}x}{c\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}\,c\left(b c^3-a d^3\right)^{4/3}}-\frac{b\,c^2\,\text{ArcTan}\left[\frac{1-\frac{2 d\left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\left(b c^3-a d^3\right)^{4/3}}+\frac{b\,c^2\,\text{Log}\left[c^3+d^3 x^3\right]}{6\left(b c^3-a d^3\right)^{4/3}}+\frac{a\,d^3\,\text{Log}\left[c^3+d^3 x^3\right]}{9\,c\left(b c^3-a d^3\right)^{4/3}}+\frac{\left(3 b c^3-2 a d^3\right)\text{Log}\left[c^3+d^3 x^3\right]}{18\,c\left(b c^3-a d^3\right)^{4/3}}-\frac{a\,d^3\,\text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3}x}{c}-\left(a+b x^3\right)^{1/3}\right]}{3\,c\left(b c^3-a d^3\right)^{4/3}}-\frac{\left(3 b c^3-2 a d^3\right)\text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3}x}{c}-\left(a+b x^3\right)^{1/3}\right]}{6\,c\left(b c^3-a d^3\right)^{4/3}}-\frac{b\,c^2\,\text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d\left(a+b x^3\right)^{1/3}\right]}{2\left(b c^3-a d^3\right)^{4/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x)^2\left(a+b x^3\right)^{1/3}},x\right]$$

Problem 35: Unable to integrate problem.

$$\int \frac{1}{(c+d\,x)^3\,\left(a+b\,x^3\right)^{1/3}}\,dx$$

Optimal (type 6, 1513 leaves, 32 steps):

$$\frac{3\,c^4\,d^2\,\left(a+b\,x^3\right)^{2/3}}{2\,\left(b\,c^3-a\,d^3\right)\,\left(c^3+d^3\,x^3\right)^2}-\frac{3\,c^3\,d^3\,x\,\left(a+b\,x^3\right)^{2/3}}{2\,\left(b\,c^3-a\,d^3\right)\,\left(c^3+d^3\,x^3\right)^2}+\frac{4\,b\,c^4\,d^2\,\left(a+b\,x^3\right)^{2/3}}{3\,\left(b\,c^3-a\,d^3\right)^2\,\left(c^3+d^3\,x^3\right)}-\frac{c\,d^2\,\left(b\,c^3-3\,a\,d^3\right)\,\left(a+b\,x^3\right)^{2/3}}{3\,\left(b\,c^3-a\,d^3\right)^2\,\left(c^3+d^3\,x^3\right)}+$$

$$\frac{d^3\,\left(3\,b\,c^3-7\,a\,d^3\right)\,x\,\left(a+b\,x^3\right)^{2/3}}{18\,\left(b\,c^3-a\,d^3\right)^2\,\left(c^3+d^3\,x^3\right)}-\frac{d^3\,\left(9\,b\,c^3-5\,a\,d^3\right)\,x\,\left(a+b\,x^3\right)^{2/3}}{18\,\left(b\,c^3-a\,d^3\right)^2\,\left(c^3+d^3\,x^3\right)}-\frac{7\,d^3\,\left(3\,b\,c^3+a\,d^3\right)\,x\,\left(a+b\,x^3\right)^{2/3}}{18\,\left(b\,c^3-a\,d^3\right)^2\,\left(c^3+d^3\,x^3\right)}-$$

$$\frac{3\,d\,x^2\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},3,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d^3\,x^3}{c^3}\right]}{2\,c^4\,\left(a+b\,x^3\right)^{1/3}}+\frac{6\,d^4\,x^5\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},3,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d^3\,x^3}{c^3}\right]}{5\,c^7\,\left(a+b\,x^3\right)^{1/3}}+\frac{2\,a^2\,d^6\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c^3-a\,d^3\right)^{1/3}\,x}{c\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,c^2\,\left(b\,c^3-a\,d^3\right)^{7/3}}+$$

$$\frac{7\,a\,d^3\,\left(3\,b\,c^3-a\,d^3\right)\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c^3-a\,d^3\right)^{1/3}\,x}{c\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,c^2\,\left(b\,c^3-a\,d^3\right)^{7/3}}+\frac{\left(9\,b^2\,c^6-12\,a\,b\,c^3\,d^3+5\,a^2\,d^6\right)\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c^3-a\,d^3\right)^{1/3}\,x}{c\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,c^2\,\left(b\,c^3-a\,d^3\right)^{7/3}}-\frac{4\,b^2\,c^4\,\text{ArcTan}\left[\frac{1-\frac{2\,d\,\left(a+b\,x^3\right)^{1/3}}{\left(b\,c^3-a\,d^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,\left(b\,c^3-a\,d^3\right)^{7/3}}+$$

$$\frac{b\,c\,\left(b\,c^3-3\,a\,d^3\right)\,\text{ArcTan}\left[\frac{1-\frac{2\,d\,\left(a+b\,x^3\right)^{1/3}}{\left(b\,c^3-a\,d^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,\left(b\,c^3-a\,d^3\right)^{7/3}}+\frac{2\,b^2\,c^4\,\text{Log}\left[c^3+d^3\,x^3\right]}{9\,\left(b\,c^3-a\,d^3\right)^{7/3}}+\frac{a^2\,d^6\,\text{Log}\left[c^3+d^3\,x^3\right]}{27\,c^2\,\left(b\,c^3-a\,d^3\right)^{7/3}}-\frac{b\,c\,\left(b\,c^3-3\,a\,d^3\right)\,\text{Log}\left[c^3+d^3\,x^3\right]}{18\,\left(b\,c^3-a\,d^3\right)^{7/3}}+$$

$$\frac{7\,a\,d^3\,\left(3\,b\,c^3-a\,d^3\right)\,\text{Log}\left[c^3+d^3\,x^3\right]}{54\,c^2\,\left(b\,c^3-a\,d^3\right)^{7/3}}+\frac{\left(9\,b^2\,c^6-12\,a\,b\,c^3\,d^3+5\,a^2\,d^6\right)\,\text{Log}\left[c^3+d^3\,x^3\right]}{54\,c^2\,\left(b\,c^3-a\,d^3\right)^{7/3}}-\frac{a^2\,d^6\,\text{Log}\left[\frac{\left(b\,c^3-a\,d^3\right)^{1/3}\,x}{c}-\left(a+b\,x^3\right)^{1/3}\right]}{9\,c^2\,\left(b\,c^3-a\,d^3\right)^{7/3}}-$$

$$\frac{7\,a\,d^3\,\left(3\,b\,c^3-a\,d^3\right)\,\text{Log}\left[\frac{\left(b\,c^3-a\,d^3\right)^{1/3}\,x}{c}-\left(a+b\,x^3\right)^{1/3}\right]}{18\,c^2\,\left(b\,c^3-a\,d^3\right)^{7/3}}-\frac{\left(9\,b^2\,c^6-12\,a\,b\,c^3\,d^3+5\,a^2\,d^6\right)\,\text{Log}\left[\frac{\left(b\,c^3-a\,d^3\right)^{1/3}\,x}{c}-\left(a+b\,x^3\right)^{1/3}\right]}{18\,c^2\,\left(b\,c^3-a\,d^3\right)^{7/3}}-$$

$$\frac{2\,b^2\,c^4\,\text{Log}\left[\left(b\,c^3-a\,d^3\right)^{1/3}+d\,\left(a+b\,x^3\right)^{1/3}\right]}{3\,\left(b\,c^3-a\,d^3\right)^{7/3}}+\frac{b\,c\,\left(b\,c^3-3\,a\,d^3\right)\,\text{Log}\left[\left(b\,c^3-a\,d^3\right)^{1/3}+d\,\left(a+b\,x^3\right)^{1/3}\right]}{6\,\left(b\,c^3-a\,d^3\right)^{7/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d\,x)^3\,\left(a+b\,x^3\right)^{1/3}},x\right]$$

Problem 36: Result valid but suboptimal antiderivative.

$$\int \frac{(c+d\,x)^4}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 5, 306 leaves, 10 steps):

$$\frac{6\,c^2\,d^2\,\left(a+b\,x^3\right)^{1/3}}{b}+\frac{d^4\,x^2\,\left(a+b\,x^3\right)^{1/3}}{3\,b}-\frac{4\,c^3\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+\frac{2\,a\,d^4\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{5/3}}+\frac{c^4\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}+$$
$$\frac{c\,d^3\,x^4\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}-\frac{2\,c^3\,d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{b^{2/3}}+\frac{a\,d^4\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{3\,b^{5/3}}$$

Result (type 5, 416 leaves, 22 steps):

$$\frac{6\,c^2\,d^2\,\left(a+b\,x^3\right)^{1/3}}{b}+\frac{d^4\,x^2\,\left(a+b\,x^3\right)^{1/3}}{3\,b}-\frac{4\,c^3\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+\frac{2\,a\,d^4\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{5/3}}+$$
$$\frac{c^4\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}+\frac{c\,d^3\,x^4\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}-$$
$$\frac{4\,c^3\,d\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}+\frac{2\,a\,d^4\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{5/3}}+\frac{2\,c^3\,d\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}-\frac{a\,d^4\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{5/3}}$$

Problem 37: Result valid but suboptimal antiderivative.

$$\int \frac{(c+d\,x)^3}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 5, 187 leaves, 8 steps):

$$\frac{3\,c\,d^2\,\left(a+b\,x^3\right)^{1/3}}{b}+\frac{d^3\,x\,\left(a+b\,x^3\right)^{1/3}}{2\,b}-\frac{\sqrt{3}\,c^2\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{b^{2/3}}+$$
$$\frac{\left(2\,b\,c^3-a\,d^3\right)\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,b\,\left(a+b\,x^3\right)^{2/3}}-\frac{3\,c^2\,d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,b^{2/3}}$$

Result (type 5, 239 leaves, 14 steps):

$$\frac{3\,c\,d^2\,\left(a+b\,x^3\right)^{1/3}}{b}+\frac{d^3\,x\,\left(a+b\,x^3\right)^{1/3}}{2\,b}-\frac{\sqrt{3}\,c^2\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{b^{2/3}}+$$
$$\frac{\left(2\,b\,c^3-a\,d^3\right)\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,b\,\left(a+b\,x^3\right)^{2/3}}-\frac{c^2\,d\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{b^{2/3}}+\frac{c^2\,d\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{2\,b^{2/3}}$$

Problem 38: Result valid but suboptimal antiderivative.

$$\int\frac{\left(c+d\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 5, 141 leaves, 7 steps):

$$\frac{d^2\,\left(a+b\,x^3\right)^{1/3}}{b}-\frac{2\,c\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+\frac{c^2\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}-\frac{c\,d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{b^{2/3}}$$

Result (type 5, 195 leaves, 13 steps):

$$\frac{d^2\,\left(a+b\,x^3\right)^{1/3}}{b}-\frac{2\,c\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+$$
$$\frac{c^2\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}-\frac{2\,c\,d\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}+\frac{c\,d\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}$$

Problem 39: Result valid but suboptimal antiderivative.

$$\int\frac{c+d\,x}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 5, 121 leaves, 5 steps):

$$-\frac{d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+\frac{c\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}-\frac{d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,b^{2/3}}$$

Result (type 5, 172 leaves, 11 steps):

$$-\frac{d\,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3}}+\frac{c\, x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{\left(a+b x^3\right)^{2/3}}-\frac{d\,\text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\, b^{2/3}}+\frac{d\,\text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6\, b^{2/3}}$$

Problem 40: Unable to integrate problem.

$$\int \frac{1}{(c+d x) \left(a+b x^3\right)^{2/3}}\, dx$$

Optimal (type 6, 332 leaves, 10 steps):

$$\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c \left(a+b x^3\right)^{2/3}}+\frac{d\,\text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} \left(b c^3-a d^3\right)^{2/3}}-\frac{d\,\text{ArcTan}\left[\frac{1-\frac{2 d \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} \left(b c^3-a d^3\right)^{2/3}}-\frac{d\,\text{Log}\left[c^3+d^3 x^3\right]}{3 \left(b c^3-a d^3\right)^{2/3}}+\frac{d\,\text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{2 \left(b c^3-a d^3\right)^{2/3}}+\frac{d\,\text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d \left(a+b x^3\right)^{1/3}\right]}{2 \left(b c^3-a d^3\right)^{2/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x) \left(a+b x^3\right)^{2/3}},x\right]$$

Problem 41: Unable to integrate problem.

$$\int \frac{1}{(c+d x)^2 \left(a+b x^3\right)^{2/3}}\, dx$$

Optimal (type 6, 760 leaves, 18 steps):

$$\frac{c^2 d^2 \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right) \left(c^3+d^3 x^3\right)}+\frac{d^4 x^2 \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right) \left(c^3+d^3 x^3\right)}+\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c^2 \left(a+b x^3\right)^{2/3}}-$$

$$\frac{d^3 x^4 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{2 c^5 \left(a+b x^3\right)^{2/3}}+\frac{2 a d^4 \text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, c \left(b c^3-a d^3\right)^{5/3}}+\frac{2 d \left(3 b c^3-a d^3\right) \text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, c \left(b c^3-a d^3\right)^{5/3}}-$$

$$\frac{2 b c^2 d \text{ArcTan}\left[\frac{1-\frac{2 d \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} \left(b c^3-a d^3\right)^{5/3}}-\frac{b c^2 d \text{Log}\left[c^3+d^3 x^3\right]}{3 \left(b c^3-a d^3\right)^{5/3}}-\frac{a d^4 \text{Log}\left[c^3+d^3 x^3\right]}{9 c \left(b c^3-a d^3\right)^{5/3}}-\frac{d \left(3 b c^3-a d^3\right) \text{Log}\left[c^3+d^3 x^3\right]}{9 c \left(b c^3-a d^3\right)^{5/3}}+$$

$$\frac{a d^4 \text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{3 c \left(b c^3-a d^3\right)^{5/3}}+\frac{d \left(3 b c^3-a d^3\right) \text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{3 c \left(b c^3-a d^3\right)^{5/3}}+\frac{b c^2 d \text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d \left(a+b x^3\right)^{1/3}\right]}{\left(b c^3-a d^3\right)^{5/3}}$$

Result (type 8, 21 leaves, 0 steps) :

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x)^2 \left(a+b x^3\right)^{2/3}},x\right]$$

Problem 42: Unable to integrate problem.

$$\int \frac{1}{(c+d x)^3 \left(a+b x^3\right)^{2/3}}\, dx$$

Optimal (type 6, 1357 leaves, 30 steps) :

$$\frac{3 c^4 d^2 \left(a+b x^3\right)^{1/3}}{2 \left(b c^3-a d^3\right) \left(c^3+d^3 x^3\right)^2}+\frac{3 c^2 d^4 x^2 \left(a+b x^3\right)^{1/3}}{2 \left(b c^3-a d^3\right) \left(c^3+d^3 x^3\right)^2}+\frac{5 b c^4 d^2 \left(a+b x^3\right)^{1/3}}{3 \left(b c^3-a d^3\right)^2 \left(c^3+d^3 x^3\right)}-\frac{c d^2 \left(b c^3-6 a d^3\right) \left(a+b x^3\right)^{1/3}}{6 \left(b c^3-a d^3\right)^2 \left(c^3+d^3 x^3\right)}+$$

$$\frac{d^4 \left(9 b c^3-4 a d^3\right) x^2 \left(a+b x^3\right)^{1/3}}{6 c \left(b c^3-a d^3\right)^2 \left(c^3+d^3 x^3\right)}+\frac{d^4 \left(3 b c^3+2 a d^3\right) x^2 \left(a+b x^3\right)^{1/3}}{3 c \left(b c^3-a d^3\right)^2 \left(c^3+d^3 x^3\right)}+\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c^3 \left(a+b x^3\right)^{2/3}}-$$

$$\frac{7 d^3 x^4 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},3,\frac{7}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{4 c^6 \left(a+b x^3\right)^{2/3}}+\frac{d^6 x^7 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},3,\frac{10}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{7 c^9 \left(a+b x^3\right)^{2/3}}+$$

$$\frac{2 a d^4 \left(6 b c^3-a d^3\right) \text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 \left(b c^3-a d^3\right)^{8/3}}+\frac{d \left(9 b^2 c^6-6 a b c^3 d^3+2 a^2 d^6\right) \text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 \left(b c^3-a d^3\right)^{8/3}}-$$

$$\frac{10 b^2 c^4 d \text{ArcTan}\left[\frac{1-\frac{2 d \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} \left(b c^3-a d^3\right)^{8/3}}+\frac{b c d \left(b c^3-6 a d^3\right) \text{ArcTan}\left[\frac{1-\frac{2 d \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} \left(b c^3-a d^3\right)^{8/3}}-\frac{5 b^2 c^4 d \text{Log}\left[c^3+d^3 x^3\right]}{9 \left(b c^3-a d^3\right)^{8/3}}+$$

$$\frac{b c d \left(b c^3-6 a d^3\right) \text{Log}\left[c^3+d^3 x^3\right]}{18 \left(b c^3-a d^3\right)^{8/3}}-\frac{a d^4 \left(6 b c^3-a d^3\right) \text{Log}\left[c^3+d^3 x^3\right]}{9 c^2 \left(b c^3-a d^3\right)^{8/3}}-\frac{d \left(9 b^2 c^6-6 a b c^3 d^3+2 a^2 d^6\right) \text{Log}\left[c^3+d^3 x^3\right]}{18 c^2 \left(b c^3-a d^3\right)^{8/3}}+$$

$$\frac{a d^4 \left(6 b c^3-a d^3\right) \text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{3 c^2 \left(b c^3-a d^3\right)^{8/3}}+\frac{d \left(9 b^2 c^6-6 a b c^3 d^3+2 a^2 d^6\right) \text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{6 c^2 \left(b c^3-a d^3\right)^{8/3}}+$$

$$\frac{5 b^2 c^4 d \text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d \left(a+b x^3\right)^{1/3}\right]}{3 \left(b c^3-a d^3\right)^{8/3}}-\frac{b c d \left(b c^3-6 a d^3\right) \text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d \left(a+b x^3\right)^{1/3}\right]}{6 \left(b c^3-a d^3\right)^{8/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x)^3 \left(a+b x^3\right)^{2/3}},x\right]$$

Problem 197: Unable to integrate problem.

$$\int \frac{\left(d^3+e^3 x^3\right)^p}{d+e x} \, dx$$

Optimal (type 6, 135 leaves, ? steps):

$$\frac{\left(d^3+e^3 x^3\right)^p \left(1+\frac{2 (d+e x)}{\left(-3+i \sqrt{3}\right) d}\right)^{-p} \left(1-\frac{2 (d+e x)}{\left(3+i \sqrt{3}\right) d}\right)^{-p} \text{AppellF1}\left[p,-p,-p,1+p,-\frac{2 (d+e x)}{\left(-3+i \sqrt{3}\right) d},\frac{2 (d+e x)}{\left(3+i \sqrt{3}\right) d}\right]}{e p}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(d^3+e^3 x^3\right)^p}{d+e x},x\right]$$

Problem 227: Result valid but suboptimal antiderivative.

$$\int x^m \left(c \left(a+b x^2\right)^2\right)^{3/2} dx$$

Optimal (type 3, 161 leaves, 3 steps):

$$\frac{a^3 c x^{1+m} \sqrt{c \left(a+b x^2\right)^2}}{(1+m) \left(a+b x^2\right)}+\frac{3 a^2 b c x^{3+m} \sqrt{c \left(a+b x^2\right)^2}}{(3+m) \left(a+b x^2\right)}+\frac{3 a b^2 c x^{5+m} \sqrt{c \left(a+b x^2\right)^2}}{(5+m) \left(a+b x^2\right)}+\frac{b^3 c x^{7+m} \sqrt{c \left(a+b x^2\right)^2}}{(7+m) \left(a+b x^2\right)}$$

Result (type 3, 205 leaves, 4 steps):

$$\frac{a^3 c x^{1+m} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(1+m) \left(a+b x^2\right)}+\frac{3 a^2 b c x^{3+m} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(3+m) \left(a+b x^2\right)}+$$
$$\frac{3 a b^2 c x^{5+m} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(5+m) \left(a+b x^2\right)}+\frac{b^3 c x^{7+m} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(7+m) \left(a+b x^2\right)}$$

Problem 228: Result valid but suboptimal antiderivative.

$$\int x^5 \left(c \left(a+b x^2\right)^2\right)^{3/2} dx$$

Optimal (type 2, 143 leaves, 4 steps):

$$\frac{a^3 c x^6 \sqrt{c \left(a+b x^2\right)^2}}{6 \left(a+b x^2\right)}+\frac{3 a^2 b c x^8 \sqrt{c \left(a+b x^2\right)^2}}{8 \left(a+b x^2\right)}+\frac{3 a b^2 c x^{10} \sqrt{c \left(a+b x^2\right)^2}}{10 \left(a+b x^2\right)}+\frac{b^3 c x^{12} \sqrt{c \left(a+b x^2\right)^2}}{12 \left(a+b x^2\right)}$$

Result (type 2, 134 leaves, 4 steps):

$$\frac{a^2 c \left(a+b x^2\right)^3 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{8 b^3}-\frac{a c \left(a+b x^2\right)^4 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{5 b^3}+\frac{c \left(a+b x^2\right)^5 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{12 b^3}$$

Problem 229: Result valid but suboptimal antiderivative.

$$\int x^4 \left(c \left(a+b x^2\right)^2\right)^{3/2} dx$$

Optimal (type 2, 143 leaves, 3 steps):

$$\frac{a^3 c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)}+\frac{3 a^2 b c x^7 \sqrt{c \left(a+b x^2\right)^2}}{7 \left(a+b x^2\right)}+\frac{a b^2 c x^9 \sqrt{c \left(a+b x^2\right)^2}}{3 \left(a+b x^2\right)}+\frac{b^3 c x^{11} \sqrt{c \left(a+b x^2\right)^2}}{11 \left(a+b x^2\right)}$$

Result (type 2, 187 leaves, 4 steps):

$$\frac{a^3 c x^5 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{5 \left(a+b x^2\right)}+\frac{3 a^2 b c x^7 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{7 \left(a+b x^2\right)}+\frac{a b^2 c x^9 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{3 \left(a+b x^2\right)}+\frac{b^3 c x^{11} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{11 \left(a+b x^2\right)}$$

Problem 230: Result valid but suboptimal antiderivative.

$$\int x^3 \left(c \left(a+b x^2\right)^2\right)^{3/2} dx$$

Optimal (type 2, 66 leaves, 4 steps):

$$-\frac{a c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^2}}{8 b^2}+\frac{c \left(a+b x^2\right)^4 \sqrt{c \left(a+b x^2\right)^2}}{10 b^2}$$

Result (type 2, 78 leaves, 4 steps):

$$-\frac{a \left(a+b x^2\right) \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{8 b^2}+\frac{\left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{5/2}}{10 b^2 c}$$

Problem 231: Result valid but suboptimal antiderivative.

$$\int x^2 \left(c \left(a+b x^2\right)^2\right)^{3/2} dx$$

Optimal (type 2, 143 leaves, 3 steps):

$$\frac{a^3 c x^3 \sqrt{c \left(a+b x^2\right)^2}}{3 \left(a+b x^2\right)}+\frac{3 a^2 b c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)}+\frac{3 a b^2 c x^7 \sqrt{c \left(a+b x^2\right)^2}}{7 \left(a+b x^2\right)}+\frac{b^3 c x^9 \sqrt{c \left(a+b x^2\right)^2}}{9 \left(a+b x^2\right)}$$

Result (type 2, 187 leaves, 4 steps):

$$\frac{a^3 c x^3 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{3 \left(a+b x^2\right)}+\frac{3 a^2 b c x^5 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{5 \left(a+b x^2\right)}+\frac{3 a b^2 c x^7 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{7 \left(a+b x^2\right)}+\frac{b^3 c x^9 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{9 \left(a+b x^2\right)}$$

Problem 233: Result valid but suboptimal antiderivative.

$$\int \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$$

Optimal (type 2, 135 leaves, 3 steps):

$$\frac{a^3 c x \sqrt{c \left(a+b x^2\right)^2}}{a+b x^2}+\frac{a^2 b c x^3 \sqrt{c \left(a+b x^2\right)^2}}{a+b x^2}+\frac{3 a b^2 c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)}+\frac{b^3 c x^7 \sqrt{c \left(a+b x^2\right)^2}}{7 \left(a+b x^2\right)}$$

Result (type 2, 175 leaves, 4 steps):

$$\frac{a^3 x \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{\left(a+b x^2\right)^3}+\frac{a^2 b x^3 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{\left(a+b x^2\right)^3}+\frac{3 a b^2 x^5 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{5 \left(a+b x^2\right)^3}+\frac{b^3 x^7 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{7 \left(a+b x^2\right)^3}$$

Problem 234: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^2\right)^{3/2}}{x} \, dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$\frac{3 a^2 b c x^2 \sqrt{c \left(a+b x^2\right)^2}}{2 \left(a+b x^2\right)}+\frac{3 a b^2 c x^4 \sqrt{c \left(a+b x^2\right)^2}}{4 \left(a+b x^2\right)}+\frac{b^3 c x^6 \sqrt{c \left(a+b x^2\right)^2}}{6 \left(a+b x^2\right)}+\frac{a^3 c \sqrt{c \left(a+b x^2\right)^2} \operatorname{Log}[x]}{a+b x^2}$$

Result (type 3, 183 leaves, 5 steps):

$$\frac{3 a^2 b c x^2 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{2 \left(a+b x^2\right)}+\frac{3 a b^2 c x^4 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{4 \left(a+b x^2\right)}+\frac{b^3 c x^6 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{6 \left(a+b x^2\right)}+\frac{a^3 c \sqrt{a^2 c+2 a b c x^2+b^2 c x^4} \operatorname{Log}[x]}{a+b x^2}$$

Problem 235: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^2\right)^{3/2}}{x^2} \, dx$$

Optimal (type 2, 134 leaves, 3 steps):

$$-\frac{a^3 c \sqrt{c \left(a+b x^2\right)^2}}{x \left(a+b x^2\right)}+\frac{3 a^2 b c x \sqrt{c \left(a+b x^2\right)^2}}{a+b x^2}+\frac{a b^2 c x^3 \sqrt{c \left(a+b x^2\right)^2}}{a+b x^2}+\frac{b^3 c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)}$$

Result (type 2, 178 leaves, 4 steps):

$$-\frac{a^3 c \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{x \left(a+b x^2\right)}+\frac{3 a^2 b c x \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{a+b x^2}+\frac{a b^2 c x^3 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{a+b x^2}+\frac{b^3 c x^5 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{5 \left(a+b x^2\right)}$$

Problem 236: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^2\right)^{3/2}}{x^3} \, dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$-\frac{a^3 c \sqrt{c \left(a+b x^2\right)^2}}{2 x^2 \left(a+b x^2\right)}+\frac{3 a b^2 c x^2 \sqrt{c \left(a+b x^2\right)^2}}{2 \left(a+b x^2\right)}+\frac{b^3 c x^4 \sqrt{c \left(a+b x^2\right)^2}}{4 \left(a+b x^2\right)}+\frac{3 a^2 b c \sqrt{c \left(a+b x^2\right)^2} \, \text{Log}[x]}{a+b x^2}$$

Result (type 3, 184 leaves, 5 steps):

$$-\frac{a^3 c \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{2 x^2 \left(a+b x^2\right)}+\frac{3 a b^2 c x^2 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{2 \left(a+b x^2\right)}+\frac{b^3 c x^4 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{4 \left(a+b x^2\right)}+\frac{3 a^2 b c \sqrt{a^2 c+2 a b c x^2+b^2 c x^4} \, \text{Log}[x]}{a+b x^2}$$

Problem 237: Result valid but suboptimal antiderivative.

$$\int x^2 \left(c \left(a+b x^2\right)^3\right)^{3/2} \, dx$$

Optimal (type 3, 253 leaves, 8 steps):

$$\frac{7}{128} a^3 c x^3 \sqrt{c \left(a+b x^2\right)^3}+\frac{21 a^5 c x \sqrt{c \left(a+b x^2\right)^3}}{1024 b \left(a+b x^2\right)}+\frac{21 a^4 c x^3 \sqrt{c \left(a+b x^2\right)^3}}{512 \left(a+b x^2\right)}+\frac{21}{320} a^2 c x^3 \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$

$$\frac{3}{40} a c x^3 \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{12} c x^3 \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}-\frac{21 a^{9/2} c \sqrt{c \left(a+b x^2\right)^3} \, \text{ArcSinh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{1024 b^{3/2} \left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 254 leaves, 9 steps):

$$\frac{7}{128} a^3 c x^3 \sqrt{c \left(a+b x^2\right)^3}+\frac{21 a^5 c x \sqrt{c \left(a+b x^2\right)^3}}{1024 b \left(a+b x^2\right)}+\frac{21 a^4 c x^3 \sqrt{c \left(a+b x^2\right)^3}}{512 \left(a+b x^2\right)}+\frac{21}{320} a^2 c x^3 \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$

$$\frac{3}{40} a c x^3 \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{12} c x^3 \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}-\frac{21 a^6 c \sqrt{c \left(a+b x^2\right)^3} \text{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a+b x^2}}\right]}{1024 b^{3/2} \left(a+b x^2\right)^{3/2}}$$

Problem 239: Result valid but suboptimal antiderivative.

$$\int \left(c \left(a+b x^2\right)^3\right)^{3/2} dx$$

Optimal (type 3, 207 leaves, 7 steps):

$$\frac{21}{128} a^3 c x \sqrt{c \left(a+b x^2\right)^3}+\frac{63 a^4 c x \sqrt{c \left(a+b x^2\right)^3}}{256 \left(a+b x^2\right)}+\frac{21}{160} a^2 c x \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$

$$\frac{9}{80} a c x \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{10} c x \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}+\frac{63 a^{7/2} c \sqrt{c \left(a+b x^2\right)^3} \text{ArcSinh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{256 \sqrt{b} \left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 208 leaves, 8 steps):

$$\frac{21}{128} a^3 c x \sqrt{c \left(a+b x^2\right)^3}+\frac{63 a^4 c x \sqrt{c \left(a+b x^2\right)^3}}{256 \left(a+b x^2\right)}+\frac{21}{160} a^2 c x \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$

$$\frac{9}{80} a c x \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{10} c x \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}+\frac{63 a^5 c \sqrt{c \left(a+b x^2\right)^3} \text{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a+b x^2}}\right]}{256 \sqrt{b} \left(a+b x^2\right)^{3/2}}$$

Problem 240: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^3\right)^{3/2}}{x} dx$$

Optimal (type 3, 192 leaves, 9 steps):

$$\frac{1}{3} a^3 c \sqrt{c \left(a+b x^2\right)^3}+\frac{a^4 c \sqrt{c \left(a+b x^2\right)^3}}{a+b x^2}+\frac{1}{5} a^2 c \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$

$$\frac{1}{7} a c \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{9} c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}-\frac{a^3 c \sqrt{c \left(a+b x^2\right)^3} \text{ArcTanh}\left[\sqrt{1+\frac{b x^2}{a}}\right]}{\left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 194 leaves, 9 steps):

$$\frac{1}{3} a^3 c \sqrt{c \left(a+b x^2\right)^3}+\frac{a^4 c \sqrt{c \left(a+b x^2\right)^3}}{a+b x^2}+\frac{1}{5} a^2 c \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$

$$\frac{1}{7} a c \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{9} c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}-\frac{a^{9/2} c \sqrt{c \left(a+b x^2\right)^3} \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right]}{\left(a+b x^2\right)^{3/2}}$$

Problem 241: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^3\right)^{3/2}}{x^2} \, dx$$

Optimal (type 3, 208 leaves, 7 steps):

$$\frac{105}{64} a^2 b c x \sqrt{c \left(a+b x^2\right)^3}+\frac{315 a^3 b c x \sqrt{c \left(a+b x^2\right)^3}}{128 \left(a+b x^2\right)}+\frac{21}{16} a b c x \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$

$$\frac{9}{8} b c x \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}-\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}}{x}+\frac{315 a^{5/2} \sqrt{b} c \sqrt{c \left(a+b x^2\right)^3} \text{ArcSinh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{128 \left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 209 leaves, 8 steps):

$$\frac{105}{64} a^2 b c x \sqrt{c \left(a+b x^2\right)^3}+\frac{315 a^3 b c x \sqrt{c \left(a+b x^2\right)^3}}{128 \left(a+b x^2\right)}+\frac{21}{16} a b c x \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$

$$\frac{9}{8} b c x \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}-\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}}{x}+\frac{315 a^4 \sqrt{b} c \sqrt{c \left(a+b x^2\right)^3} \text{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a+b x^2}}\right]}{128 \left(a+b x^2\right)^{3/2}}$$

Problem 242: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^3\right)^{3/2}}{x^3} \, dx$$

Optimal (type 3, 202 leaves, 9 steps):

$$\frac{3}{2} a^2 b c \sqrt{c \left(a+b x^2\right)^3}+\frac{9 a^3 b c \sqrt{c \left(a+b x^2\right)^3}}{2 \left(a+b x^2\right)}+\frac{9}{10} a b c \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$
$$\frac{9}{14} b c \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}-\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}}{2 x^2}-\frac{9 a^2 b c \sqrt{c \left(a+b x^2\right)^3} \, \text{ArcTanh}\left[\sqrt{1+\frac{b x^2}{a}}\right]}{2 \left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 204 leaves, 9 steps):

$$\frac{3}{2} a^2 b c \sqrt{c \left(a+b x^2\right)^3}+\frac{9 a^3 b c \sqrt{c \left(a+b x^2\right)^3}}{2 \left(a+b x^2\right)}+\frac{9}{10} a b c \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+$$
$$\frac{9}{14} b c \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}-\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}}{2 x^2}-\frac{9 a^{7/2} b c \sqrt{c \left(a+b x^2\right)^3} \, \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right]}{2 \left(a+b x^2\right)^{3/2}}$$

Problem 243: Result valid but suboptimal antiderivative.

$$\int x^2 \left(\frac{c}{a+b x^2}\right)^{3/2} \, dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{c x \sqrt{\frac{c}{a+b x^2}}}{b}+\frac{\sqrt{a} \, c \sqrt{\frac{c}{a+b x^2}} \sqrt{1+\frac{b x^2}{a}} \, \text{ArcSinh}\left[\frac{\sqrt{b} \, x}{\sqrt{a}}\right]}{b^{3/2}}$$

Result (type 3, 75 leaves, 4 steps):

$$-\frac{c x \sqrt{\frac{c}{a+b x^2}}}{b}+\frac{c \sqrt{\frac{c}{a+b x^2}} \sqrt{a+b x^2} \, \text{ArcTanh}\left[\frac{\sqrt{b} \, x}{\sqrt{a+b x^2}}\right]}{b^{3/2}}$$

Problem 246: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\frac{c}{a+b x^2}\right)^{3/2}}{x} \, dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$\frac{c \sqrt{\frac{c}{a+b x^2}}}{a} - \frac{c \sqrt{\frac{c}{a+b x^2}} \sqrt{1+\frac{b x^2}{a}} \, \text{ArcTanh}\left[\sqrt{1+\frac{b x^2}{a}}\right]}{a}$$

Result (type 3, 73 leaves, 5 steps):

$$\frac{c \sqrt{\frac{c}{a+b x^2}}}{a} - \frac{c \sqrt{\frac{c}{a+b x^2}} \sqrt{a+b x^2} \, \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right]}{a^{3/2}}$$

Problem 248: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\frac{c}{a+b x^2}\right)^{3/2}}{x^3} \, dx$$

Optimal (type 3, 104 leaves, 6 steps):

$$-\frac{3 b c \sqrt{\frac{c}{a+b x^2}}}{2 a^2} - \frac{c \sqrt{\frac{c}{a+b x^2}}}{2 a x^2} + \frac{3 b c \sqrt{\frac{c}{a+b x^2}} \sqrt{1+\frac{b x^2}{a}} \, \text{ArcTanh}\left[\sqrt{1+\frac{b x^2}{a}}\right]}{2 a^2}$$

Result (type 3, 112 leaves, 6 steps):

$$\frac{c \sqrt{\frac{c}{a+b x^2}}}{a x^2} - \frac{3 c \sqrt{\frac{c}{a+b x^2}} \left(a+b x^2\right)}{2 a^2 x^2} + \frac{3 b c \sqrt{\frac{c}{a+b x^2}} \sqrt{a+b x^2} \, \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right]}{2 a^{5/2}}$$

Problem 249: Result valid but suboptimal antiderivative.

$$\int x^7 \left(c \sqrt{a+b x^2}\right)^{3/2} dx$$

Optimal (type 2, 138 leaves, 4 steps):

$$-\frac{2 a^3 \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)}{7 b^4}+\frac{6 a^2 \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)^2}{11 b^4}-\frac{2 a \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)^3}{5 b^4}+\frac{2 \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)^4}{19 b^4}$$

Result (type 2, 152 leaves, 4 steps):

$$-\frac{2 a^3 c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{3/2}}{7 b^4}+\frac{6 a^2 c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{5/2}}{11 b^4}-\frac{2 a c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{7/2}}{5 b^4}+\frac{2 c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{9/2}}{19 b^4}$$

Problem 250: Result valid but suboptimal antiderivative.

$$\int x^5 \left(c \sqrt{a+b x^2}\right)^{3/2} dx$$

Optimal (type 2, 102 leaves, 4 steps):

$$\frac{2 a^2 \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)}{7 b^3}-\frac{4 a \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)^2}{11 b^3}+\frac{2 \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)^3}{15 b^3}$$

Result (type 2, 113 leaves, 4 steps):

$$\frac{2 a^2 c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{3/2}}{7 b^3}-\frac{4 a c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{5/2}}{11 b^3}+\frac{2 c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{7/2}}{15 b^3}$$

Problem 251: Result valid but suboptimal antiderivative.

$$\int x^3 \left(c \sqrt{a+b x^2}\right)^{3/2} dx$$

Optimal (type 2, 66 leaves, 4 steps):

$$-\frac{2 a \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)}{7 b^2}+\frac{2 \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)^2}{11 b^2}$$

Result (type 2, 74 leaves, 4 steps):

$$-\frac{2 a c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{3/2}}{7 b^2}+\frac{2 c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{5/2}}{11 b^2}$$

Problem 253: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{x} \, dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\frac{2}{3} \left(c \sqrt{a+b x^2}\right)^{3/2}+\frac{\left(c \sqrt{a+b x^2}\right)^{3/2} \text{ArcTan}\left[\left(1+\frac{b x^2}{a}\right)^{1/4}\right]}{\left(1+\frac{b x^2}{a}\right)^{3/4}}-\frac{\left(c \sqrt{a+b x^2}\right)^{3/2} \text{ArcTanh}\left[\left(1+\frac{b x^2}{a}\right)^{1/4}\right]}{\left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 3, 141 leaves, 7 steps):

$$\frac{2}{3} c \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2}+\frac{a^{3/4} c \sqrt{c \sqrt{a+b x^2}} \text{ArcTan}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right]}{\left(a+b x^2\right)^{1/4}}-\frac{a^{3/4} c \sqrt{c \sqrt{a+b x^2}} \text{ArcTanh}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right]}{\left(a+b x^2\right)^{1/4}}$$

Problem 254: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{x^3} \, dx$$

Optimal (type 3, 133 leaves, 7 steps):

$$-\frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{2 x^2}+\frac{3 b \left(c \sqrt{a+b x^2}\right)^{3/2} \text{ArcTan}\left[\left(1+\frac{b x^2}{a}\right)^{1/4}\right]}{4 a \left(1+\frac{b x^2}{a}\right)^{3/4}}-\frac{3 b \left(c \sqrt{a+b x^2}\right)^{3/2} \text{ArcTanh}\left[\left(1+\frac{b x^2}{a}\right)^{1/4}\right]}{4 a \left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 3, 151 leaves, 7 steps):

$$-\frac{c \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2}}{2 x^2}+\frac{3 b c \sqrt{c \sqrt{a+b x^2}} \text{ArcTan}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right]}{4 a^{1/4} \left(a+b x^2\right)^{1/4}}-\frac{3 b c \sqrt{c \sqrt{a+b x^2}} \text{ArcTanh}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right]}{4 a^{1/4} \left(a+b x^2\right)^{1/4}}$$

Problem 255: Result valid but suboptimal antiderivative.

$$\int x^2 \left(c \sqrt{a+b x^2}\right)^{3/2} \, dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$\frac{2 a x \left(c \sqrt{a+b x^2}\right)^{3/2}}{15 b}+\frac{2}{9} x^3 \left(c \sqrt{a+b x^2}\right)^{3/2}-\frac{4 a^2 x \left(c \sqrt{a+b x^2}\right)^{3/2}}{15 b \left(a+b x^2\right)}+\frac{4 a^{3/2} \left(c \sqrt{a+b x^2}\right)^{3/2} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{15 b^{3/2} \left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 4, 191 leaves, 6 steps):

$$-\frac{4 a^2 c x \sqrt{c \sqrt{a+b x^2}}}{15 b \sqrt{a+b x^2}}+\frac{2 a c x \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2}}{15 b}+$$
$$\frac{2}{9} c x^3 \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2}+\frac{4 a^{5/2} c \sqrt{c \sqrt{a+b x^2}} \left(1+\frac{b x^2}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{15 b^{3/2} \sqrt{a+b x^2}}$$

Problem 256: Result valid but suboptimal antiderivative.

$$\int \left(c \sqrt{a+b x^2}\right)^{3/2} dx$$

Optimal (type 4, 119 leaves, 4 steps):

$$\frac{2}{5} x \left(c \sqrt{a+b x^2}\right)^{3/2}+\frac{6 a x \left(c \sqrt{a+b x^2}\right)^{3/2}}{5 \left(a+b x^2\right)}-\frac{6 \sqrt{a} \left(c \sqrt{a+b x^2}\right)^{3/2} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{5 \sqrt{b} \left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 4, 146 leaves, 5 steps):

$$\frac{6 a c x \sqrt{c \sqrt{a+b x^2}}}{5 \sqrt{a+b x^2}}+\frac{2}{5} c x \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2}-\frac{6 a^{3/2} c \sqrt{c \sqrt{a+b x^2}} \left(1+\frac{b x^2}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{5 \sqrt{b} \sqrt{a+b x^2}}$$

Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{x^2} dx$$

Optimal (type 4, 115 leaves, 4 steps):

$$-\frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{x}+\frac{3 b x \left(c \sqrt{a+b x^2}\right)^{3/2}}{a+b x^2}-\frac{3 \sqrt{b} \left(c \sqrt{a+b x^2}\right)^{3/2} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right],2\right]}{\sqrt{a} \left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 4, 142 leaves, 5 steps):

$$\frac{3\,b\,c\,x\sqrt{c\sqrt{a+b\,x^2}}}{\sqrt{a+b\,x^2}}-\frac{c\sqrt{c\sqrt{a+b\,x^2}}\,\sqrt{a+b\,x^2}}{x}-\frac{3\sqrt{a}\,\sqrt{b}\,c\sqrt{c\sqrt{a+b\,x^2}}\,\left(1+\frac{b\,x^2}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{\sqrt{a+b\,x^2}}$$

Problem 258: Result valid but suboptimal antiderivative.

$$\int\frac{\left(c\sqrt{a+b\,x^2}\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 154 leaves, 5 steps):

$$-\frac{\left(c\sqrt{a+b\,x^2}\right)^{3/2}}{3\,x^3}-\frac{b\left(c\sqrt{a+b\,x^2}\right)^{3/2}}{2\,a\,x}+\frac{b^2\,x\left(c\sqrt{a+b\,x^2}\right)^{3/2}}{2\,a\left(a+b\,x^2\right)}-\frac{b^{3/2}\left(c\sqrt{a+b\,x^2}\right)^{3/2}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{2\,a^{3/2}\left(1+\frac{b\,x^2}{a}\right)^{3/4}}$$

Result (type 4, 193 leaves, 6 steps):

$$\frac{b^2\,c\,x\sqrt{c\sqrt{a+b\,x^2}}}{2\,a\sqrt{a+b\,x^2}}-\frac{c\sqrt{c\sqrt{a+b\,x^2}}\,\sqrt{a+b\,x^2}}{3\,x^3}-\frac{b\,c\sqrt{c\sqrt{a+b\,x^2}}\,\sqrt{a+b\,x^2}}{2\,a\,x}-\frac{b^{3/2}\,c\sqrt{c\sqrt{a+b\,x^2}}\,\left(1+\frac{b\,x^2}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{2\sqrt{a}\,\sqrt{a+b\,x^2}}$$

Problem 318: Result valid but suboptimal antiderivative.

$$\int x^5\sqrt{a+\frac{b}{c+d\,x^2}}\,dx$$

Optimal (type 3, 216 leaves, 7 steps):

$$-\frac{\left(b^2+4\,a\,b\,c-8\,a^2\,c^2\right)\left(c+d\,x^2\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{16\,a^2\,d^3}-\frac{\left(b+4\,a\,c\right)\left(c+d\,x^2\right)^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{8\,a\,d^3}+$$

$$\frac{\left(c+d\,x^2\right)^3\left(\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}\right)^{3/2}}{6\,a\,d^3}+\frac{b\left(b^2+4\,a\,b\,c+8\,a^2\,c^2\right)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{a}}\right]}{16\,a^{5/2}\,d^3}$$

Result (type 3, 259 leaves, 9 steps):

$$\frac{\left(b^2+4 a b c+8 a^2 c^2\right) \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}{16 a^2 d^3}-\frac{\left(3 b+8 a c\right) \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}} \left(b+a \left(c+d x^2\right)\right)}{24 a^2 d^3}+$$

$$\frac{x^2 \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}} \left(b+a \left(c+d x^2\right)\right)}{6 a d^2}+\frac{b \left(b^2+4 a b c+8 a^2 c^2\right) \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{16 a^{5/2} d^3 \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 319: Result valid but suboptimal antiderivative.

$$\int x^3 \sqrt{a+\frac{b}{c+d x^2}}\, dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$\frac{\left(b-4 a c\right) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 a d^2}+\frac{\left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 d^2}-\frac{b \left(b+4 a c\right) \text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{8 a^{3/2} d^2}$$

Result (type 3, 181 leaves, 8 steps):

$$-\frac{\left(b+4 a c\right) \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}{8 a d^2}+\frac{\left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}} \left(b+a \left(c+d x^2\right)\right)}{4 a d^2}-\frac{b \left(b+4 a c\right) \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{8 a^{3/2} d^2 \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 321: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d x^2}}}{x}\, dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\sqrt{a}\, \text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]-\frac{\sqrt{b+a c}\, \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{\sqrt{c}}$$

Result (type 3, 184 leaves, 9 steps):

$$\frac{\sqrt{a}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{c+d\,x^2}}{\sqrt{b+a\,(c+d\,x^2)}}\right]}{\sqrt{b+a\,(c+d\,x^2)}}-\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}$$

Problem 322: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d\,x^2}}}{x^3}\,dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$-\frac{(c+d\,x^2)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{2\,c\,x^2}+\frac{b\,d\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{2\,c^{3/2}\,\sqrt{b+a\,c}}$$

Result (type 3, 140 leaves, 6 steps):

$$-\frac{(c+d\,x^2)\,\sqrt{a+\frac{b}{c+d\,x^2}}}{2\,c\,x^2}+\frac{b\,d\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{2\,c^{3/2}\,\sqrt{b+a\,c}\,\sqrt{b+a\,(c+d\,x^2)}}$$

Problem 323: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d\,x^2}}}{x^5}\,dx$$

Optimal (type 3, 174 leaves, 6 steps):

$$\frac{(5\,b+4\,a\,c)\,d\,(c+d\,x^2)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{8\,c^2\,(b+a\,c)\,x^2}-\frac{(c+d\,x^2)^2\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{4\,c^2\,x^4}-\frac{b\,(3\,b+4\,a\,c)\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{8\,c^{5/2}\,(b+a\,c)^{3/2}}$$

Result (type 3, 218 leaves, 7 steps):

$$\frac{(3b+4ac)\,d\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2}-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{4c(b+ac)x^4}-\frac{b(3b+4ac)d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\,\text{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right]}{8c^{5/2}(b+ac)^{3/2}\sqrt{b+a(c+dx^2)}}$$

Problem 324: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx$$

Optimal (type 3, 265 leaves, 7 steps):

$$-\frac{(11b^2+20abc+8a^2c^2)\,d^2\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16c^3(b+ac)^2x^2}+\frac{(3b+4ac)\,d\,(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3(b+ac)x^4}-\frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6c^2(b+ac)x^6}+\frac{b(5b^2+12abc+8a^2c^2)\,d^3\,\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{16c^{7/2}(b+ac)^{5/2}}$$

Result (type 3, 271 leaves, 9 steps):

$$-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6}+\frac{(5b+4ac)\,d\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4}-\frac{(5b+2ac)(3b+4ac)\,d^2\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{48c^3(b+ac)^2x^2}+\frac{b(5b^2+12abc+8a^2c^2)\,d^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\,\text{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right]}{16c^{7/2}(b+ac)^{5/2}\sqrt{b+a(c+dx^2)}}$$

Problem 325: Result valid but suboptimal antiderivative.

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

Optimal (type 4, 368 leaves, 8 steps):

$$-\frac{\left(2 b^2+7 a b c-3 a^2 c^2\right) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 a^2 d^2}+\frac{(b-3 a c) x \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 a d^2}+\frac{x^3 \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d}+$$

$$\frac{\sqrt{c} \left(2 b^2+7 a b c-3 a^2 c^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 a^2 d^{5/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}-\frac{c^{3/2} (b-3 a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 a d^{5/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}$$

Result (type 4, 478 leaves, 8 steps):

$$-\frac{\left(2 b^2+7 a b c-3 a^2 c^2\right) x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 a^2 d^2 \sqrt{b+a \left(c+d x^2\right)}}+\frac{(b-3 a c) x \left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 a d^2 \sqrt{b+a \left(c+d x^2\right)}}+$$

$$\frac{x^3 \left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{5 d \sqrt{b+a \left(c+d x^2\right)}}+\frac{\sqrt{c} \left(2 b^2+7 a b c-3 a^2 c^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 a^2 d^{5/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}}-$$

$$\frac{c^{3/2} (b-3 a c) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 a d^{5/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 326: Result valid but suboptimal antiderivative.

$$\int x^2 \sqrt{a+\frac{b}{c+d x^2}} \, dx$$

Optimal (type 4, 282 leaves, 7 steps):

$$\frac{(b-a c)\,x\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 a d}+\frac{x\,(c+d x^2)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 d}-$$

$$\frac{\sqrt{c}\,(b-a c)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 a d^{3/2}\sqrt{\frac{c\,(b+a c+a d x^2)}{(b+a c)\,(c+d x^2)}}}-\frac{c^{3/2}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 d^{3/2}\sqrt{\frac{c\,(b+a c+a d x^2)}{(b+a c)\,(c+d x^2)}}}$$

Result (type 4, 370 leaves, 7 steps):

$$\frac{(b-a c)\,x\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{3 a d\sqrt{b+a\,(c+d x^2)}}+\frac{x\,(c+d x^2)\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{3 d\sqrt{b+a\,(c+d x^2)}}-$$

$$\frac{\sqrt{c}\,(b-a c)\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 a d^{3/2}\sqrt{\frac{c\,(b+a c+a d x^2)}{(b+a c)\,(c+d x^2)}}\sqrt{b+a\,(c+d x^2)}}-\frac{c^{3/2}\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 d^{3/2}\sqrt{\frac{c\,(b+a c+a d x^2)}{(b+a c)\,(c+d x^2)}}\sqrt{b+a\,(c+d x^2)}}$$

Problem 327: Result valid but suboptimal antiderivative.

$$\int\sqrt{a+\frac{b}{c+d x^2}}\,dx$$

Optimal (type 4, 213 leaves, 6 steps):

$$x\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}-\frac{\sqrt{c}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{\sqrt{d}\sqrt{\frac{c\,(b+a c+a d x^2)}{(b+a c)\,(c+d x^2)}}}+\frac{\sqrt{c}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{\sqrt{d}\sqrt{\frac{c\,(b+a c+a d x^2)}{(b+a c)\,(c+d x^2)}}}$$

Result (type 4, 279 leaves, 6 steps):

$$\frac{x\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{\sqrt{b+a\left(c+d x^2\right)}}-\frac{\sqrt{c}\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{\sqrt{d}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{b+a\left(c+d x^2\right)}}+$$

$$\frac{\sqrt{c}\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{\sqrt{d}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{b+a\left(c+d x^2\right)}}$$

Problem 328: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{a+\frac{b}{c+d x^2}}}{x^2}\,dx$$

Optimal (type 4, 265 leaves, 8 steps):

$$\frac{d x\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c}-\frac{\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x}-$$

$$\frac{\sqrt{d}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{\sqrt{c}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}+\frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{(b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 353 leaves, 8 steps):

$$\frac{d x\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{c\sqrt{b+a\left(c+d x^2\right)}}-\frac{\left(c+d x^2\right)\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{c x\sqrt{b+a\left(c+d x^2\right)}}-$$

$$\frac{\sqrt{d}\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{\sqrt{c}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{b+a\left(c+d x^2\right)}}+\frac{a\sqrt{c}\sqrt{d}\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{(b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{b+a\left(c+d x^2\right)}}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d x^2}}}{x^4}\,dx$$

Optimal (type 4, 362 leaves, 8 steps):

$$-\frac{(2 b+a c) d^2 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 c^2 (b+a c)}-\frac{\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 c x^3}+\frac{(2 b+a c) d \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 c^2 (b+a c) x}+$$
$$\frac{(2 b+a c) d^{3/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 c^{3/2} (b+a c) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}-\frac{a d^{3/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 \sqrt{c}\, (b+a c) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}$$

Result (type 4, 472 leaves, 8 steps):

$$-\frac{(2 b+a c) d^2 x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{3 c^2 (b+a c) \sqrt{b+a \left(c+d x^2\right)}}-\frac{\left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{3 c x^3 \sqrt{b+a \left(c+d x^2\right)}}+\frac{(2 b+a c) d \left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{3 c^2 (b+a c) x \sqrt{b+a \left(c+d x^2\right)}}+$$
$$\frac{(2 b+a c) d^{3/2} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 c^{3/2} (b+a c) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}}-\frac{a d^{3/2} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 \sqrt{c}\, (b+a c) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 330: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d x^2}}}{x^6}\,dx$$

Optimal (type 4, 466 leaves, 9 steps):

$$\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^3 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 c^3 (b+a c)^2}-\frac{\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c x^5}+$$

$$\frac{(4 b+3 a c) d \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 c^2 (b+a c) x^3}-\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^2 \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 c^3 (b+a c)^2 x}-$$

$$\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 c^{5/2} (b+a c)^2 \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}+\frac{a (4 b+3 a c) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 c^{3/2} (b+a c)^2 \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}$$

Result (type 4, 598 leaves, 9 steps):

$$\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^3 x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 c^3 (b+a c)^2 \sqrt{b+a \left(c+d x^2\right)}}-\frac{\left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{5 c x^5 \sqrt{b+a \left(c+d x^2\right)}}+$$

$$\frac{(4 b+3 a c) d \left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 c^2 (b+a c) x^3 \sqrt{b+a \left(c+d x^2\right)}}-\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^2 \left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 c^3 (b+a c)^2 x \sqrt{b+a \left(c+d x^2\right)}}-$$

$$\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^{5/2} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 c^{5/2} (b+a c)^2 \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}}+$$

$$\frac{a (4 b+3 a c) d^{5/2} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 c^{3/2} (b+a c)^2 \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 331: Result valid but suboptimal antiderivative.

$$\int x^5 \left(a+\frac{b}{c+d x^2}\right)^{3/2} dx$$

Optimal (type 3, 249 leaves, 8 steps):

$$-\frac{b c^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d^3}-\frac{\left(5 b^2+60 a b c-24 a^2 c^2\right) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{48 a d^3}-\frac{(b+12 a c) \left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{24 d^3}+\frac{\left(c+d x^2\right)^3 \left(\frac{b+a c+a d x^2}{c+d x^2}\right)^{5/2}}{6 a d^3}-\frac{b \left(b^2+12 a b c-24 a^2 c^2\right) \text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{16 a^{3/2} d^3}$$

Result (type 3, 311 leaves, 10 steps):

$$-\frac{\left(b^2+12 a b c-24 a^2 c^2\right) \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}{16 a d^3}-\frac{\left(b^2+12 a b c-24 a^2 c^2\right) \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}} \left(b+a \left(c+d x^2\right)\right)}{24 a b d^3}-\frac{c^2 \sqrt{a+\frac{b}{c+d x^2}} \left(b+a \left(c+d x^2\right)\right)^2}{b d^3}+\frac{\left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}} \left(b+a \left(c+d x^2\right)\right)^2}{6 a d^3}-\frac{b \left(b^2+12 a b c-24 a^2 c^2\right) \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{16 a^{3/2} d^3 \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 332: Result valid but suboptimal antiderivative.

$$\int x^3 \left(a+\frac{b}{c+d x^2}\right)^{3/2} dx$$

Optimal (type 3, 172 leaves, 7 steps):

$$\frac{b c \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d^2}+\frac{(5 b-4 a c) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 d^2}+\frac{a \left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 d^2}+\frac{3 b (b-4 a c) \text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{8 \sqrt{a} d^2}$$

Result (type 3, 222 leaves, 9 steps):

$$\frac{3 (b-4 a c) \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}{8 d^2}+\frac{(b-4 a c) \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}} \left(b+a \left(c+d x^2\right)\right)}{4 b d^2}+\frac{c \sqrt{a+\frac{b}{c+d x^2}} \left(b+a \left(c+d x^2\right)\right)^2}{b d^2}+\frac{3 b (b-4 a c) \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{8 \sqrt{a} d^2 \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 334: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}{x} \, dx$$

Optimal (type 3, 126 leaves, 7 steps):

$$\frac{b \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c}+a^{3/2} \text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]-\frac{(b+a c)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{c^{3/2}}$$

Result (type 3, 206 leaves, 10 steps):

$$\frac{b \sqrt{a+\frac{b}{c+d x^2}}}{c}+\frac{a^{3/2} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{\sqrt{b+a \left(c+d x^2\right)}}-\frac{(b+a c)^{3/2} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}\right]}{c^{3/2} \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 335: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}{x^3} \, dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$-\frac{3 b d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{2 c^2}-\frac{\left(c+d x^2\right) \left(\frac{b+a c+a d x^2}{c+d x^2}\right)^{3/2}}{2 c x^2}+\frac{3 b \sqrt{b+a c}\, d \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{2 c^{5/2}}$$

Result (type 3, 170 leaves, 7 steps):

$$-\frac{3 b d \sqrt{a+\frac{b}{c+d x^2}}}{2 c^2}-\frac{\sqrt{a+\frac{b}{c+d x^2}} \left(b+a \left(c+d x^2\right)\right)}{2 c x^2}+\frac{3 b \sqrt{b+a c}\, d \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}\right]}{2 c^{5/2} \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 336: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}{x^5} \, dx$$

Optimal (type 3, 205 leaves, 7 steps):

$$\frac{b\,d^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c^3}+\frac{(9\,b+4\,a\,c)\,d\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{8\,c^3\,x^2}-\frac{(b+a\,c)\,(c+d\,x^2)^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{4\,c^3\,x^4}-\frac{3\,b\,(5\,b+4\,a\,c)\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{8\,c^{7/2}\sqrt{b+a\,c}}$$

Result (type 3, 260 leaves, 8 steps):

$$\frac{3\,b\,(5\,b+4\,a\,c)\,d^2\sqrt{a+\frac{b}{c+d\,x^2}}}{8\,c^3\,(b+a\,c)}+\frac{(5\,b+4\,a\,c)\,d\sqrt{a+\frac{b}{c+d\,x^2}}\,(b+a\,(c+d\,x^2))}{8\,c^2\,(b+a\,c)\,x^2}-$$

$$\frac{\sqrt{a+\frac{b}{c+d\,x^2}}\,(b+a\,(c+d\,x^2))^2}{4\,c\,(b+a\,c)\,x^4}-\frac{3\,b\,(5\,b+4\,a\,c)\,d^2\sqrt{c+d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\sqrt{c+d\,x^2}}{\sqrt{c}\sqrt{b+a\,(c+d\,x^2)}}\right]}{8\,c^{7/2}\sqrt{b+a\,c}\sqrt{b+a\,(c+d\,x^2)}}$$

Problem 337: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}{x^7}\,dx$$

Optimal (type 3, 292 leaves, 8 steps):

$$-\frac{b\,d^3\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c^4}-\frac{(79\,b^2+108\,a\,b\,c+24\,a^2\,c^2)\,d^2\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{48\,c^4\,(b+a\,c)\,x^2}+$$

$$\frac{(11\,b+12\,a\,c)\,d\,(c+d\,x^2)^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{24\,c^4\,x^4}-\frac{(c+d\,x^2)^3\left(\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}\right)^{5/2}}{6\,c^2\,(b+a\,c)\,x^6}+\frac{b\,(35\,b^2+60\,a\,b\,c+24\,a^2\,c^2)\,d^3\,\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{16\,c^{9/2}\,(b+a\,c)^{3/2}}$$

Result (type 3, 287 leaves, 10 steps):

$$-\frac{\left(105 b^2+110 a b c+8 a^2 c^2\right) d^3 \sqrt{a+\frac{b}{c+d x^2}}}{48 c^4 (b+a c)}-\frac{(b+a c) \sqrt{a+\frac{b}{c+d x^2}}}{6 c x^6}+\frac{7 b d \sqrt{a+\frac{b}{c+d x^2}}}{24 c^2 x^4}-$$

$$\frac{b (35 b+32 a c) d^2 \sqrt{a+\frac{b}{c+d x^2}}}{48 c^3 (b+a c) x^2}+\frac{b \left(35 b^2+60 a b c+24 a^2 c^2\right) d^3 \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}\right]}{16 c^{9/2} (b+a c)^{3/2} \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 338: Result valid but suboptimal antiderivative.

$$\int x^4 \left(a+\frac{b}{c+d x^2}\right)^{3/2} dx$$

Optimal (type 4, 405 leaves, 9 steps):

$$\frac{\left(b^2-14 a b c+a^2 c^2\right) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 a d^2}+\frac{(7 b-a c) x \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d^2}+\frac{6 a x^3 \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d}-\frac{x^3 \left(b+a c+a d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d}-$$

$$\frac{\sqrt{c} \left(b^2-14 a b c+a^2 c^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 a d^{5/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}-\frac{c^{3/2} (7 b-a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 d^{5/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}$$

Result (type 4, 526 leaves, 9 steps):

$$\frac{\left(b^2-14\,a\,b\,c+a^2\,c^2\right)x\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{5\,a\,d^2\sqrt{b+a\left(c+d\,x^2\right)}}+\frac{\left(7\,b-a\,c\right)x\left(c+d\,x^2\right)\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{5\,d^2\sqrt{b+a\left(c+d\,x^2\right)}}+$$

$$\frac{6\,a\,x^3\left(c+d\,x^2\right)\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{5\,d\sqrt{b+a\left(c+d\,x^2\right)}}-\frac{x^3\left(b+a\,c+a\,d\,x^2\right)^{3/2}\sqrt{a+\frac{b}{c+d\,x^2}}}{d\sqrt{b+a\left(c+d\,x^2\right)}}-$$

$$\frac{\sqrt{c}\left(b^2-14\,a\,b\,c+a^2\,c^2\right)\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{5\,a\,d^{5/2}\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{\left(b+a\,c\right)\left(c+d\,x^2\right)}}\,\sqrt{b+a\left(c+d\,x^2\right)}}-$$

$$\frac{c^{3/2}\left(7\,b-a\,c\right)\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{5\,d^{5/2}\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{\left(b+a\,c\right)\left(c+d\,x^2\right)}}\,\sqrt{b+a\left(c+d\,x^2\right)}}$$

Problem 339: Result valid but suboptimal antiderivative.

$$\int x^2\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}dx$$

Optimal (type 4, 331 leaves, 8 steps):

$$\frac{\left(7\,b-a\,c\right)x\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{3\,d}+\frac{4\,a\,x\left(c+d\,x^2\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{3\,d}-\frac{x\left(b+a\,c+a\,d\,x^2\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{d}-$$

$$\frac{\sqrt{c}\left(7\,b-a\,c\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{3\,d^{3/2}\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{\left(b+a\,c\right)\left(c+d\,x^2\right)}}}+\frac{\sqrt{c}\left(3\,b-a\,c\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{3\,d^{3/2}\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{\left(b+a\,c\right)\left(c+d\,x^2\right)}}}$$

Result (type 4, 430 leaves, 8 steps):

$$\frac{(7b-ac)\,x\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\,(c+dx^2)}}+\frac{4ax\,(c+dx^2)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\,(c+dx^2)}}-$$

$$\frac{x\,(b+ac+adx^2)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\,(c+dx^2)}}-\frac{\sqrt{c}\,(7b-ac)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{3d^{3/2}\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}\,\sqrt{b+a\,(c+dx^2)}}+$$

$$\frac{\sqrt{c}\,(3b-ac)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{3d^{3/2}\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}\,\sqrt{b+a\,(c+dx^2)}}$$

Problem 340: Result valid but suboptimal antiderivative.

$$\int\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

Optimal (type 4, 260 leaves, 7 steps):

$$\frac{bx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c}-\frac{(b-ac)\,x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c}+$$

$$\frac{(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}+\frac{a\sqrt{c}\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{\sqrt{d}\,\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}$$

Result (type 4, 348 leaves, 7 steps):

$$\frac{b\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{c\,\sqrt{b+a\,\left(c+d\,x^2\right)}}-\frac{(b-a\,c)\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{c\,\sqrt{b+a\,\left(c+d\,x^2\right)}}+$$

$$\frac{(b-a\,c)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}+\frac{a\,\sqrt{c}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{\sqrt{d}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}$$

Problem 341: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}{x^2}\,dx$$

Optimal (type 4, 312 leaves, 8 steps):

$$\frac{b\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c\,x}+\frac{(2\,b+a\,c)\,d\,x\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c^2}-\frac{(2\,b+a\,c)\,\left(c+d\,x^2\right)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c^2\,x}-$$

$$\frac{(2\,b+a\,c)\,\sqrt{d}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{c^{3/2}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}}+\frac{a\,\sqrt{d}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{\sqrt{c}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}}$$

Result (type 4, 422 leaves, 8 steps):

$$\frac{b\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{c\,x\,\sqrt{b+a\,\left(c+d\,x^2\right)}}+\frac{(2\,b+a\,c)\,d\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{c^2\,\sqrt{b+a\,\left(c+d\,x^2\right)}}-\frac{(2\,b+a\,c)\,\left(c+d\,x^2\right)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{c^2\,x\,\sqrt{b+a\,\left(c+d\,x^2\right)}}-$$

$$\frac{(2\,b+a\,c)\,\sqrt{d}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{c^{3/2}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}+\frac{a\,\sqrt{d}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{\sqrt{c}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}$$

Problem 342: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 388 leaves, 9 steps):

$$\frac{b\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x^3}-\frac{(8 b+a c) d^2 x\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 c^3}-\frac{(4 b+a c)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 c^2 x^3}+\frac{(8 b+a c) d\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 c^3 x}+\frac{(8 b+a c) d^{3/2}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 c^{5/2}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}-\frac{a(4 b+a c) d^{3/2}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 c^{3/2}(b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 520 leaves, 9 steps):

$$\frac{b\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{c x^3\sqrt{b+a\left(c+d x^2\right)}}-\frac{(8 b+a c) d^2 x\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{3 c^3\sqrt{b+a\left(c+d x^2\right)}}-\frac{(4 b+a c)\left(c+d x^2\right)\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{3 c^2 x^3\sqrt{b+a\left(c+d x^2\right)}}+\frac{(8 b+a c) d\left(c+d x^2\right)\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{3 c^3 x\sqrt{b+a\left(c+d x^2\right)}}+\frac{(8 b+a c) d^{3/2}\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 c^{5/2}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{b+a\left(c+d x^2\right)}}-\frac{a(4 b+a c) d^{3/2}\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 c^{3/2}(b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{b+a\left(c+d x^2\right)}}$$

Problem 343: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}{x^6}\,dx$$

Optimal (type 4, 494 leaves, 10 steps):

$$\frac{b\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x^5}+\frac{\left(16 b^2+16 a b c+a^2 c^2\right) d^3 x\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^4 (b+a c)}-\frac{(6 b+a c)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^2 x^5}+\frac{(8 b+a c) d\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^3 x^3}-\frac{\left(16 b^2+16 a b c+a^2 c^2\right) d^2\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^4 (b+a c) x}-\frac{\left(16 b^2+16 a b c+a^2 c^2\right) d^{5/2}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{5 c^{7/2} (b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}+\frac{a (8 b+a c) d^{5/2}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{5 c^{5/2} (b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 648 leaves, 10 steps):

$$\frac{b\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{c x^5\sqrt{b+a\left(c+d x^2\right)}}+\frac{\left(16 b^2+16 a b c+a^2 c^2\right) d^3 x\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{5 c^4 (b+a c)\sqrt{b+a\left(c+d x^2\right)}}-\frac{(6 b+a c)\left(c+d x^2\right)\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{5 c^2 x^5\sqrt{b+a\left(c+d x^2\right)}}+\frac{(8 b+a c) d\left(c+d x^2\right)\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{5 c^3 x^3\sqrt{b+a\left(c+d x^2\right)}}-\frac{\left(16 b^2+16 a b c+a^2 c^2\right) d^2\left(c+d x^2\right)\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}{5 c^4 (b+a c) x\sqrt{b+a\left(c+d x^2\right)}}-\frac{\left(16 b^2+16 a b c+a^2 c^2\right) d^{5/2}\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{5 c^{7/2} (b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{b+a\left(c+d x^2\right)}}+\frac{a (8 b+a c) d^{5/2}\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{5 c^{5/2} (b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{b+a\left(c+d x^2\right)}}$$

Problem 344: Result valid but suboptimal antiderivative.

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+d x^2}}}\,dx$$

Optimal (type 3, 225 leaves, 7 steps):

$$\frac{\left(5 b^2+12 a b c+8 a^2 c^2\right) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{16 a^3 d^3}-\frac{(5 b+8 a c) \left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{24 a^2 d^3}+\frac{x^2 \left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{6 a d^2}-\frac{b \left(5 b^2+12 a b c+8 a^2 c^2\right) \text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{16 a^{7/2} d^3}$$

Result (type 3, 267 leaves, 9 steps):

$$\frac{\left(5 b^2+12 a b c+8 a^2 c^2\right) \left(b+a \left(c+d x^2\right)\right)}{16 a^3 d^3 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{(5 b+8 a c) \left(c+d x^2\right) \left(b+a \left(c+d x^2\right)\right)}{24 a^2 d^3 \sqrt{a+\frac{b}{c+d x^2}}}+\frac{x^2 \left(c+d x^2\right) \left(b+a \left(c+d x^2\right)\right)}{6 a d^2 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{b \left(5 b^2+12 a b c+8 a^2 c^2\right) \sqrt{b+a \left(c+d x^2\right)} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{16 a^{7/2} d^3 \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 345: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+d x^2}}}\,dx$$

Optimal (type 3, 148 leaves, 6 steps):

$$-\frac{(3 b+4 a c) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 a^2 d^2}+\frac{\left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 a d^2}+\frac{b (3 b+4 a c) \text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{8 a^{5/2} d^2}$$

Result (type 3, 189 leaves, 8 steps):

$$-\frac{(3b+4ac)\left(b+a\left(c+dx^2\right)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{\left(c+dx^2\right)\left(b+a\left(c+dx^2\right)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}}+\frac{b(3b+4ac)\sqrt{b+a\left(c+dx^2\right)}\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right]}{8a^{5/2}d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 347: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{\sqrt{a}}-\frac{\sqrt{c}\,\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{\sqrt{b+ac}}$$

Result (type 3, 184 leaves, 9 steps):

$$\frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right]}{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}\,\text{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right]}{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 348: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

Optimal (type 3, 108 leaves, 5 steps):

$$-\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2(b+ac)x^2}-\frac{b\,d\,\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{2\sqrt{c}(b+ac)^{3/2}}$$

Result (type 3, 148 leaves, 6 steps):

$$-\frac{b+a\left(c+d x^2\right)}{2 (b+a c) x^2 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{b d \sqrt{b+a\left(c+d x^2\right)}\ \text{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a\left(c+d x^2\right)}}\right]}{2 \sqrt{c}\ (b+a c)^{3/2} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 349: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5 \sqrt{a+\frac{b}{c+d x^2}}}\, dx$$

Optimal (type 3, 177 leaves, 6 steps):

$$\frac{(b+4 a c)\, d\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 c (b+a c)^2 x^2}-\frac{\left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 c (b+a c) x^4}+\frac{b (b+4 a c)\, d^2\, \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{8 c^{3/2} (b+a c)^{5/2}}$$

Result (type 3, 218 leaves, 7 steps):

$$\frac{(b+4 a c)\, d\left(b+a\left(c+d x^2\right)\right)}{8 c (b+a c)^2 x^2 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{\left(c+d x^2\right)\left(b+a\left(c+d x^2\right)\right)}{4 c (b+a c) x^4 \sqrt{a+\frac{b}{c+d x^2}}}+\frac{b (b+4 a c)\, d^2 \sqrt{b+a\left(c+d x^2\right)}\ \text{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a\left(c+d x^2\right)}}\right]}{8 c^{3/2} (b+a c)^{5/2} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 350: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+d x^2}}}\, dx$$

Optimal (type 4, 443 leaves, 8 steps):

$$-\frac{(4b+3ac)\,x\,(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{x^3\,(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(8b^2+13abc+3a^2c^2)\,x\,(b+ac+adx^2)}{15a^3d^2\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-$$

$$\frac{\sqrt{c}\,(8b^2+13abc+3a^2c^2)\,(b+ac+adx^2)\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{15a^3d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}+\frac{c^{3/2}\,(4b+3ac)\,(b+ac+adx^2)\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{15a^2d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}$$

Result (type 4, 498 leaves, 8 steps):

$$-\frac{(4b+3ac)\,x\sqrt{b+ac+adx^2}\,\sqrt{b+a\,(c+dx^2)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}}+$$

$$\frac{x^3\sqrt{b+ac+adx^2}\,\sqrt{b+a\,(c+dx^2)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}}+\frac{(8b^2+13abc+3a^2c^2)\,x\sqrt{b+ac+adx^2}\,\sqrt{b+a\,(c+dx^2)}}{15a^3d^2\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}-$$

$$\frac{\sqrt{c}\,(8b^2+13abc+3a^2c^2)\sqrt{b+ac+adx^2}\,\sqrt{b+a\,(c+dx^2)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{15a^3d^{5/2}\,(c+dx^2)\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}+$$

$$\frac{c^{3/2}\,(4b+3ac)\sqrt{b+ac+adx^2}\,\sqrt{b+a\,(c+dx^2)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{15a^2d^{5/2}\,(c+dx^2)\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 351: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

Optimal (type 4, 354 leaves, 7 steps):

$$\frac{x\left(b+a c+a d x^2\right)}{3 a d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(2 b+a c)\, x\left(b+a c+a d x^2\right)}{3 a^2 d\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+$$
$$\frac{\sqrt{c}\,(2 b+a c)\left(b+a c+a d x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 a^2 d^{3/2}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}-\frac{c^{3/2}\left(b+a c+a d x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 a d^{3/2}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 398 leaves, 7 steps):

$$\frac{x \sqrt{b+a c+a d x^2} \sqrt{b+a\left(c+d x^2\right)}}{3 a d \sqrt{a+\frac{b}{c+d x^2}}}-\frac{(2 b+a c)\, x \sqrt{b+a c+a d x^2} \sqrt{b+a\left(c+d x^2\right)}}{3 a^2 d\left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}+$$
$$\frac{\sqrt{c}\,(2 b+a c) \sqrt{b+a c+a d x^2} \sqrt{b+a\left(c+d x^2\right)}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 a^2 d^{3/2}\left(c+d x^2\right) \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}} \sqrt{a+\frac{b}{c+d x^2}}}-$$
$$\frac{c^{3/2} \sqrt{b+a c+a d x^2} \sqrt{b+a\left(c+d x^2\right)}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 a d^{3/2}\left(c+d x^2\right) \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 352: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+d x^2}}}\, dx$$

Optimal (type 4, 286 leaves, 6 steps):

$$\frac{x\left(b+a c+a d x^2\right)}{a\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{\sqrt{c}\left(b+a c+a d x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{a \sqrt{d}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}+\frac{c^{3/2}\left(b+a c+a d x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{(b+a c) \sqrt{d}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 319 leaves, 6 steps):

$$\frac{x \sqrt{b+a c+a d x^2} \sqrt{b+a \left(c+d x^2\right)}}{a \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}-\frac{\sqrt{c} \sqrt{b+a c+a d x^2} \sqrt{b+a \left(c+d x^2\right)} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{a \sqrt{d} \left(c+d x^2\right) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{c^{3/2} \sqrt{b+a c+a d x^2} \sqrt{b+a \left(c+d x^2\right)} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{(b+a c) \sqrt{d} \left(c+d x^2\right) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 353: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 \sqrt{a+\frac{b}{c+d x^2}}} \, dx$$

Optimal (type 4, 343 leaves, 8 steps) :

$$-\frac{b+a c+a d x^2}{(b+a c) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{d x \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-$$

$$\frac{\sqrt{c} \sqrt{d} \left(b+a c+a d x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{(b+a c) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}+\frac{\sqrt{c} \sqrt{d} \left(b+a c+a d x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{(b+a c) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}$$

Result (type 4, 387 leaves, 8 steps) :

$$-\frac{\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{(b+a c)\,x\,\sqrt{a+\frac{b}{c+d x^2}}}+\frac{d\,x\,\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{(b+a c)\left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{(b+a c)\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\,\sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{(b+a c)\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\,\sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 354: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x^4\sqrt{a+\frac{b}{c+d x^2}}}\,dx$$

Optimal (type 4, 431 leaves, 8 steps):

$$-\frac{b+a c+a d x^2}{3\,(b+a c)\,x^3\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(b-a c)\,d\left(b+a c+a d x^2\right)}{3\,c\,(b+a c)^2\,x\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{(b-a c)\,d^2\,x\left(b+a c+a d x^2\right)}{3\,c\,(b+a c)^2\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-$$

$$\frac{(b-a c)\,d^{3/2}\left(b+a c+a d x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3\sqrt{c}\,(b+a c)^2\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}-\frac{a\sqrt{c}\,d^{3/2}\left(b+a c+a d x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3\,(b+a c)^2\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 486 leaves, 8 steps):

$$-\frac{\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{3\,(b+a c)\,x^3\,\sqrt{a+\frac{b}{c+d x^2}}}-\frac{(b-a c)\,d\,\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{3\,c\,(b+a c)^2\,x\,\sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{(b-a c)\,d^2\,x\,\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{3\,c\,(b+a c)^2\left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}-\frac{(b-a c)\,d^{3/2}\,\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3\sqrt{c}\,(b+a c)^2\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\,\sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\frac{a\sqrt{c}\,d^{3/2}\,\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3\,(b+a c)^2\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\,\sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 355: Result valid but suboptimal antiderivative.

$$\int \frac{x^5}{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}\,dx$$

Optimal (type 3, 310 leaves, 8 steps):

$$-\frac{(b+a c)^2\left(c+d x^2\right)^3}{a\,b^2\,d^3\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{\left(35 b^2+60 a b c+24 a^2 c^2\right)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{16\,a^4\,d^3}-\frac{\left(35 b^2+60 a b c+24 a^2 c^2\right)\left(c+d x^2\right)^2\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{24\,a^3\,b\,d^3}+$$

$$\frac{\left(7 b^2+12 a b c+6 a^2 c^2\right)\left(c+d x^2\right)^3\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{6\,a^2\,b^2\,d^3}-\frac{b\left(35 b^2+60 a b c+24 a^2 c^2\right)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{16\,a^{9/2}\,d^3}$$

Result (type 3, 323 leaves, 10 steps):

$$\frac{(b+a c)^2 \left(c+d x^2\right)^2}{a^2 b d^3 \sqrt{a+\frac{b}{c+d x^2}}}+\frac{\left(35 b^2+60 a b c+24 a^2 c^2\right) \left(b+a \left(c+d x^2\right)\right)}{16 a^4 d^3 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{\left(35 b^2+60 a b c+24 a^2 c^2\right) \left(c+d x^2\right) \left(b+a \left(c+d x^2\right)\right)}{24 a^3 b d^3 \sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{\left(c+d x^2\right)^2 \left(b+a \left(c+d x^2\right)\right)}{6 a^2 d^3 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{b \left(35 b^2+60 a b c+24 a^2 c^2\right) \sqrt{b+a \left(c+d x^2\right)} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{16 a^{9/2} d^3 \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 356: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\left(a+\frac{b}{c+d x^2}\right)^{3/2}} \, dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$-\frac{b (b+a c)}{a^3 d^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(7 b+4 a c) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 a^3 d^2}+\frac{\left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 a^2 d^2}+\frac{3 b (5 b+4 a c) \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{8 a^{7/2} d^2}$$

Result (type 3, 242 leaves, 9 steps):

$$-\frac{(b+a c) \left(c+d x^2\right)^2}{a b d^2 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{3 (5 b+4 a c) \left(b+a \left(c+d x^2\right)\right)}{8 a^3 d^2 \sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{(5 b+4 a c) \left(c+d x^2\right) \left(b+a \left(c+d x^2\right)\right)}{4 a^2 b d^2 \sqrt{a+\frac{b}{c+d x^2}}}+\frac{3 b (5 b+4 a c) \sqrt{b+a \left(c+d x^2\right)} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{8 a^{7/2} d^2 \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 358: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x \left(a+\frac{b}{c+d x^2}\right)^{3/2}} \, dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$-\frac{b}{a\,(b+a\,c)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{a}}\right]}{a^{3/2}}-\frac{c^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{(b+a\,c)^{3/2}}$$

Result (type 3, 214 leaves, 10 steps):

$$-\frac{b}{a\,(b+a\,c)\,\sqrt{a+\frac{b}{c+d\,x^2}}}+\frac{\sqrt{b+a\,(c+d\,x^2)}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{c+d\,x^2}}{\sqrt{b+a\,(c+d\,x^2)}}\right]}{a^{3/2}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}-\frac{c^{3/2}\,\sqrt{b+a\,(c+d\,x^2)}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{(b+a\,c)^{3/2}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}$$

Problem 359: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3\,\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}\,dx$$

Optimal (type 3, 146 leaves, 6 steps):

$$\frac{3\,b\,d}{2\,(b+a\,c)^2\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-\frac{c+d\,x^2}{2\,(b+a\,c)\,x^2\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-\frac{3\,b\,\sqrt{c}\,d\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{2\,(b+a\,c)^{5/2}}$$

Result (type 3, 174 leaves, 7 steps):

$$\frac{3\,b\,d}{2\,(b+a\,c)^2\,\sqrt{a+\frac{b}{c+d\,x^2}}}-\frac{c+d\,x^2}{2\,(b+a\,c)\,x^2\,\sqrt{a+\frac{b}{c+d\,x^2}}}-\frac{3\,b\,\sqrt{c}\,d\,\sqrt{b+a\,(c+d\,x^2)}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{2\,(b+a\,c)^{5/2}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}$$

Problem 360: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5\,\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}\,dx$$

Optimal (type 3, 212 leaves, 7 steps):

$$-\frac{a\,b\,d^2}{(b+a\,c)^3\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-\frac{(3\,b-4\,a\,c)\,d\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{8\,(b+a\,c)^3\,x^2}-\frac{(c+d\,x^2)^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{4\,(b+a\,c)^2\,x^4}-\frac{3\,b\,(b-4\,a\,c)\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{8\,\sqrt{c}\,(b+a\,c)^{7/2}}$$

Result (type 3, 246 leaves, 8 steps):

$$\frac{3\,b\,(b-4\,a\,c)\,d^2}{8\,c\,(b+a\,c)^3\sqrt{a+\frac{b}{c+d\,x^2}}}-\frac{(b-4\,a\,c)\,d\,(c+d\,x^2)}{8\,c\,(b+a\,c)^2\,x^2\sqrt{a+\frac{b}{c+d\,x^2}}}-\frac{(c+d\,x^2)^2}{4\,c\,(b+a\,c)\,x^4\sqrt{a+\frac{b}{c+d\,x^2}}}-\frac{3\,b\,(b-4\,a\,c)\,d^2\sqrt{b+a\,(c+d\,x^2)}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{8\,\sqrt{c}\,(b+a\,c)^{7/2}\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}$$

Problem 361: Result valid but suboptimal antiderivative.

$$\int\frac{x^4}{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}\,dx$$

Optimal (type 4, 482 leaves, 9 steps):

$$-\frac{x^3\,(c+d\,x^2)}{a\,d\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-\frac{(8\,b+a\,c)\,x\,(b+a\,c+a\,d\,x^2)}{5\,a^3\,d^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{6\,x^3\,(b+a\,c+a\,d\,x^2)}{5\,a^2\,d\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{(16\,b^2+16\,a\,b\,c+a^2\,c^2)\,x\,(b+a\,c+a\,d\,x^2)}{5\,a^4\,d^2\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-$$

$$\frac{\sqrt{c}\,(16\,b^2+16\,a\,b\,c+a^2\,c^2)\,(b+a\,c+a\,d\,x^2)\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{5\,a^4\,d^{5/2}\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}+\frac{c^{3/2}\,(8\,b+a\,c)\,(b+a\,c+a\,d\,x^2)\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{5\,a^3\,d^{5/2}\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}$$

Result (type 4, 559 leaves, 9 steps):

$$-\frac{x^3\,(c+d\,x^2)\,\sqrt{b+a\,(c+d\,x^2)}}{a\,d\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}-\frac{(8\,b+a\,c)\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,(c+d\,x^2)}}{5\,a^3\,d^2\,\sqrt{a+\frac{b}{c+d\,x^2}}}+$$

$$\frac{6\,x^3\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,(c+d\,x^2)}}{5\,a^2\,d\,\sqrt{a+\frac{b}{c+d\,x^2}}}+\frac{(16\,b^2+16\,a\,b\,c+a^2\,c^2)\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,(c+d\,x^2)}}{5\,a^4\,d^2\,(c+d\,x^2)\,\sqrt{a+\frac{b}{c+d\,x^2}}}-$$

$$\frac{\sqrt{c}\,(16\,b^2+16\,a\,b\,c+a^2\,c^2)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,(c+d\,x^2)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{5\,a^4\,d^{5/2}\,(c+d\,x^2)\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}}+$$

$$\frac{c^{3/2}\,(8\,b+a\,c)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,(c+d\,x^2)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{5\,a^3\,d^{5/2}\,(c+d\,x^2)\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}}$$

Problem 362: Result valid but suboptimal antiderivative.

$$\int\frac{x^2}{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}\,dx$$

Optimal (type 4, 409 leaves, 8 steps):

$$-\frac{x\,(c+d\,x^2)}{a\,d\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{4\,x\,(b+a\,c+a\,d\,x^2)}{3\,a^2\,d\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-\frac{(8\,b+a\,c)\,x\,(b+a\,c+a\,d\,x^2)}{3\,a^3\,d\,(c+d\,x^2)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+$$

$$\frac{\sqrt{c}\,(8\,b+a\,c)\,(b+a\,c+a\,d\,x^2)\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{3\,a^3\,d^{3/2}\,(c+d\,x^2)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}-\frac{c^{3/2}\,(4\,b+a\,c)\,(b+a\,c+a\,d\,x^2)\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{3\,a^2\,(b+a\,c)\,d^{3/2}\,(c+d\,x^2)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}$$

Result (type 4, 475 leaves, 8 steps):

$$-\frac{x\left(c+d x^2\right)\sqrt{b+a\left(c+d x^2\right)}}{a d\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}+\frac{4 x\sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}}{3 a^2 d\sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\frac{(8 b+a c) x\sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}}{3 a^3 d\left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}+\frac{\sqrt{c}\,(8 b+a c)\sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 a^3 d^{3/2}\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\frac{c^{3/2}(4 b+a c)\sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 a^2 (b+a c) d^{3/2}\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 363: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}\,dx$$

Optimal (type 4, 356 leaves, 7 steps):

$$-\frac{b x}{a (b+a c)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{(2 b+a c) x\left(b+a c+a d x^2\right)}{a^2 (b+a c)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-$$

$$\frac{\sqrt{c}\,(2 b+a c)\left(b+a c+a d x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{a^2 (b+a c)\sqrt{d}\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}+\frac{c^{3/2}\left(b+a c+a d x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{a (b+a c)\sqrt{d}\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 411 leaves, 7 steps):

$$-\frac{b\,x\,\sqrt{b+a\left(c+d\,x^2\right)}}{a\,(b+a\,c)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}+\frac{(2\,b+a\,c)\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\left(c+d\,x^2\right)}}{a^2\,(b+a\,c)\,\left(c+d\,x^2\right)\,\sqrt{a+\frac{b}{c+d\,x^2}}}-$$

$$\frac{\sqrt{c}\,(2\,b+a\,c)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\left(c+d\,x^2\right)}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{a^2\,(b+a\,c)\,\sqrt{d}\,\left(c+d\,x^2\right)\,\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\left(c+d\,x^2\right)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}}+$$

$$\frac{c^{3/2}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\left(c+d\,x^2\right)}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{a\,(b+a\,c)\,\sqrt{d}\,\left(c+d\,x^2\right)\,\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\left(c+d\,x^2\right)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}}$$

Problem 364: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x^2\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}\,dx$$

Optimal (type 4, 410 leaves, 8 steps):

$$-\frac{b}{a\,(b+a\,c)\,x\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{(b-a\,c)\left(b+a\,c+a\,d\,x^2\right)}{a\,(b+a\,c)^2\,x\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-\frac{(b-a\,c)\,d\,x\left(b+a\,c+a\,d\,x^2\right)}{a\,(b+a\,c)^2\left(c+d\,x^2\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+$$

$$\frac{\sqrt{c}\,(b-a\,c)\,\sqrt{d}\left(b+a\,c+a\,d\,x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{a\,(b+a\,c)^2\left(c+d\,x^2\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\left(c+d\,x^2\right)}}}+\frac{c^{3/2}\,\sqrt{d}\left(b+a\,c+a\,d\,x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{(b+a\,c)^2\left(c+d\,x^2\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\left(c+d\,x^2\right)}}}$$

Result (type 4, 476 leaves, 8 steps):

$$-\frac{b\sqrt{b+a\left(c+d x^2\right)}}{a\,(b+a c)\,x\sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}+\frac{(b-a c)\sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}}{a\,(b+a c)^2 x\sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\frac{(b-a c)\,d\,x\sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}}{a\,(b+a c)^2\left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}+\frac{\sqrt{c}\,(b-a c)\sqrt{d}\sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{a\,(b+a c)^2\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{c^{3/2}\sqrt{d}\sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{(b+a c)^2\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 365: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x^4\left(a+\frac{b}{c+d x^2}\right)^{3/2}}\,dx$$

Optimal (type 4, 490 leaves, 9 steps):

$$-\frac{b}{a\,(b+a c)\,x^3\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{(3 b-a c)\left(b+a c+a d x^2\right)}{3 a\,(b+a c)^2 x^3\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(7 b-a c)\,d\left(b+a c+a d x^2\right)}{3\,(b+a c)^3 x\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{(7 b-a c)\,d^2 x\left(b+a c+a d x^2\right)}{3\,(b+a c)^3\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-$$

$$\frac{\sqrt{c}\,(7 b-a c)\,d^{3/2}\left(b+a c+a d x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3\,(b+a c)^3\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}+\frac{\sqrt{c}\,(3 b-a c)\,d^{3/2}\left(b+a c+a d x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3\,(b+a c)^3\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 567 leaves, 9 steps):

$$-\frac{b\sqrt{b+a\left(c+d x^2\right)}}{a (b+a c) x^3 \sqrt{b+a c+a d x^2}\sqrt{a+\frac{b}{c+d x^2}}}+\frac{(3 b-a c) \sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}}{3 a (b+a c)^2 x^3 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{(7 b-a c) d \sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}}{3 (b+a c)^3 x \sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{(7 b-a c) d^2 x \sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}}{3 (b+a c)^3 \left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}-\frac{\sqrt{c}\,(7 b-a c) d^{3/2} \sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 (b+a c)^3 \left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{\sqrt{c}\,(3 b-a c) d^{3/2} \sqrt{b+a c+a d x^2}\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 (b+a c)^3 \left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 396: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(\frac{\sqrt{a x^{2 n}}}{\sqrt{1+x^n}}+\frac{2 x^{-n}\sqrt{a x^{2 n}}}{(2+n)\sqrt{1+x^n}}\right)dx$$

Optimal (type 3, 34 leaves, ? steps):

$$\frac{2 x^{1-n}\sqrt{a x^{2 n}}\sqrt{1+x^n}}{2+n}$$

Result (type 5, 80 leaves, 5 steps) :

$$\frac{x\sqrt{a x^{2 n}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right]}{1+n}+\frac{2 x^{1-n}\sqrt{a x^{2 n}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-x^n\right]}{2+n}$$

Problem 616: Unable to integrate problem.

$$\int\frac{1}{x^2}\left(a+b x+c x^2\right)^m\left(d+e x+f x^2+g x^3\right)^n\left(-a d+(b d m+a e n) x+(c d+b e+a f+2 c d m+b e m+b e n+2 a f n) x^2+\right.$$
$$\left.(2 c e+2 b f+2 a g+2 c e m+b f m+c e n+2 b f n+3 a g n) x^3+(3 c f+3 b g+2 c f m+b g m+2 c f n+3 b g n) x^4+c g (4+2 m+3 n) x^5\right)dx$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{\left(a+b x+c x^2\right)^{1+m}\left(d+e x+f x^2+g x^3\right)^{1+n}}{x}$$

Result (type 8, 306 leaves, 2 steps):

$$(c (d+2 d m)+b e (1+m+n)+a f (1+2 n)) \text{CannotIntegrate}\left[\left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n, x\right]-a d \text{CannotIntegrate}\left[\frac{\left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n}{x^2}, x\right]+(b d m+a e n) \text{CannotIntegrate}\left[\frac{\left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n}{x}, x\right]+(c e (2+2 m+n)+b f (2+m+2 n)+a g (2+3 n)) \text{CannotIntegrate}\left[x \left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n, x\right]+(c f (3+2 m+2 n)+b g (3+m+3 n)) \text{CannotIntegrate}\left[x^2 \left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n, x\right]+c g (4+2 m+3 n) \text{CannotIntegrate}\left[x^3 \left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n, x\right]$$

Problem 617: Unable to integrate problem.

$$\int \frac{1}{x^3}\left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n \left(-2 a d+(-b d-a e+b d m+a e n) x+(2 c d m+b e m+b e n+2 a f n) x^2+(c e+b f+a g+2 c e m+b f m+c e n+2 b f n+3 a g n) x^3+(2 c f+2 b g+2 c f m+b g m+2 c f n+3 b g n) x^4+c g (3+2 m+3 n) x^5\right) dx$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{\left(a+b x+c x^2\right)^{1+m} \left(d+e x+f x^2+g x^3\right)^{1+n}}{x^2}$$

Result (type 8, 305 leaves, 2 steps):

$$(c e (1+2 m+n)+b f (1+m+2 n)+a g (1+3 n)) \text{CannotIntegrate}\left[\left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n, x\right]-2 a d \text{CannotIntegrate}\left[\frac{\left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n}{x^3}, x\right]-(b d (1-m)+a e (1-n)) \text{CannotIntegrate}\left[\frac{\left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n}{x^2}, x\right]+(2 c d m+2 a f n+b e (m+n)) \text{CannotIntegrate}\left[\frac{\left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n}{x}, x\right]+(2 c f (1+m+n)+b g (2+m+3 n)) \text{CannotIntegrate}\left[x \left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n, x\right]+c g (3+2 m+3 n) \text{CannotIntegrate}\left[x^2 \left(a+b x+c x^2\right)^m \left(d+e x+f x^2+g x^3\right)^n, x\right]$$

Problem 852: Result valid but suboptimal antiderivative.

$$\int \sqrt{\frac{1}{-1+x^2}}\, dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\sqrt{1-x^2}\,\sqrt{\frac{1}{-1+x^2}}\,\text{ArcSin}[x]$$

Result (type 3, 33 leaves, 3 steps):

$$\sqrt{\frac{1}{-1+x^2}}\,\sqrt{-1+x^2}\,\text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 853: Result valid but suboptimal antiderivative.

$$\int\sqrt{\frac{1+x^2}{-1+x^4}}\,dx$$

Optimal (type 3, 25 leaves, 3 steps):

$$\sqrt{1-x^2}\,\sqrt{\frac{1}{-1+x^2}}\,\text{ArcSin}[x]$$

Result (type 3, 33 leaves, 4 steps):

$$\sqrt{\frac{1}{-1+x^2}}\,\sqrt{-1+x^2}\,\text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 941: Result unnecessarily involves higher level functions.

$$\int\left(\left(1-x^6\right)^{2/3}+\frac{\left(1-x^6\right)^{2/3}}{x^6}\right)dx$$

Optimal (type 2, 35 leaves, ? steps):

$$-\frac{\left(1-x^6\right)^{2/3}}{5\,x^5}+\frac{1}{5}\,x\,\left(1-x^6\right)^{2/3}$$

Result (type 5, 36 leaves, 3 steps):

$$-\frac{\text{Hypergeometric2F1}\left[-\frac{5}{6},-\frac{2}{3},\frac{1}{6},x^6\right]}{5\,x^5}+x\,\text{Hypergeometric2F1}\left[-\frac{2}{3},\frac{1}{6},\frac{7}{6},x^6\right]$$

Problem 995: Unable to integrate problem.

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Optimal (type 3, 63 leaves, ? steps):

$$\frac{1}{4}\left(3x+\sqrt{-1+x^2}\right)\sqrt{1-x^2+x\sqrt{-1+x^2}}+\frac{3\,\text{ArcSin}\left[x-\sqrt{-1+x^2}\right]}{4\sqrt{2}}$$

Result (type 8, 24 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\sqrt{1-x^2+x\sqrt{-1+x^2}},x\right]$$

Problem 996: Unable to integrate problem.

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2}\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\sqrt{1+x}}-\frac{3\,\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]}{2\sqrt{2}}$$

Result (type 8, 31 leaves, 1 step):

$$\text{CannotIntegrate}\left[\frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}},x\right]$$

Problem 997: Result valid but suboptimal antiderivative.

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\sqrt{2\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{-2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right]+\sqrt{2\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right]$$

Result (type 3, 319 leaves, 25 steps):

$$-2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]-\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]-$$

$$\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]-2\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+$$

$$\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]-\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]$$

Problem 1016: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x\left(3+3x+x^2\right)\left(3+3x+3x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\times 3^{1/3}(1+x)}{\left(2+(1+x)^3\right)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}-\frac{\text{Log}\left[1-(1+x)^3\right]}{6\times 3^{1/3}}+\frac{\text{Log}\left[3^{1/3}(1+x)-\left(2+(1+x)^3\right)^{1/3}\right]}{2\times 3^{1/3}}$$

Result (type 3, 123 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2(1+x)}{3^{1/6}\left(2+(1+x)^3\right)^{1/3}}\right]}{3^{5/6}}+\frac{\text{Log}\left[1-\frac{3^{1/3}(1+x)}{\left(2+(1+x)^3\right)^{1/3}}\right]}{3\times 3^{1/3}}-\frac{\text{Log}\left[1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{3^{1/3}(1+x)}{\left(2+(1+x)^3\right)^{1/3}}\right]}{6\times 3^{1/3}}$$

Problem 1017: Unable to integrate problem.

$$\int\frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 103 leaves, ? steps):

$$\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}-\frac{\text{Log}\left[1+2(1-x)^3-x^3\right]}{2\times 2^{2/3}}+\frac{3\,\text{Log}\left[2^{1/3}(1-x)+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 103 leaves, 5 steps):

$$-\left(1+\mathrm{i}\sqrt{3}\right)\text{CannotIntegrate}\left[\frac{1}{\left(-1-\mathrm{i}\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}},x\right]-$$
$$\left(1-\mathrm{i}\sqrt{3}\right)\text{CannotIntegrate}\left[\frac{1}{\left(-1+\mathrm{i}\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}},x\right]-x\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right]$$

Problem 1018: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{\sqrt{-1+x^4}\left(1+x^4\right)}\,dx$$

Optimal (type 3, 49 leaves, ? steps):

$$-\frac{1}{4}\text{ArcTan}\left[\frac{1+x^2}{x\sqrt{-1+x^4}}\right]-\frac{1}{4}\text{ArcTanh}\left[\frac{1-x^2}{x\sqrt{-1+x^4}}\right]$$

Result (type 3, 47 leaves, 9 steps) :

$$\left(-\frac{1}{8}-\frac{\mathrm{i}}{8}\right)\text{ArcTan}\left[\frac{(1+\mathrm{i})\,x}{\sqrt{-1+x^4}}\right]+\left(\frac{1}{8}+\frac{\mathrm{i}}{8}\right)\text{ArcTanh}\left[\frac{(1+\mathrm{i})\,x}{\sqrt{-1+x^4}}\right]$$

Problem 1023: Unable to integrate problem.

$$\int\left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n\,dx$$

Optimal (type 3, 34 leaves, ? steps):

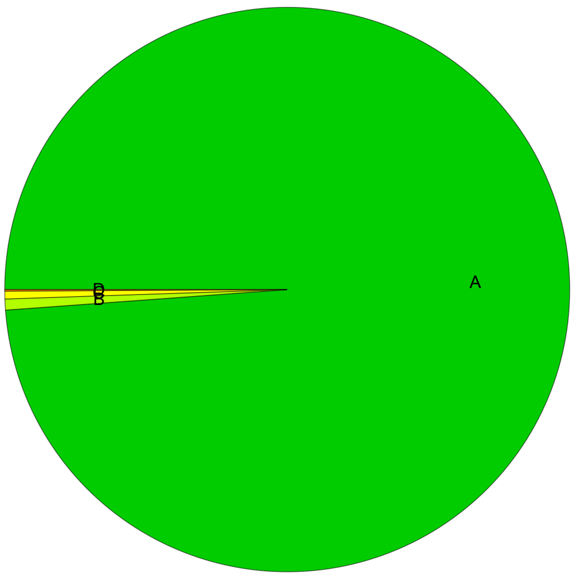
$$-\frac{(1-x)\left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n}{1+n}$$

Result (type 8, 25 leaves, 0 steps) :

$$\text{CannotIntegrate}\left[\left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n,x\right]$$

Summary of Integration Test Results

26125 integration problems



A - 25822 optimal antiderivatives

B - 164 valid but suboptimal antiderivatives

C - 116 unnecessarily complex antiderivatives

D - 23 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives